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Statistical Analysis of Kevlar 49/Epoxy Composite Stress-Rupture Data

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STATISTICAL ANALYSIS OF KEVLAR 49/EPOXY COMPOSITE STRESS-RUPTURE DATA*

Abstract

Statistical analyses are presented for LLNL stress-rupture data sets involving Kevlar 49/epoxy strands and NASA Kevlar 49/epoxy spherical pressure vessels subjected to sustained loading. Raw data, summarized inferences, and figures are included.

Introduction

Lifetime properties of Kevlar 49/epoxy composites subjected to sustained loading may be extracted from the statistical analysis of stress-rupture data on composite strands and NASA spherical pressure vessels generated at LLNL in the 1970s. The purpose of this report is to present the collection of statistical analyses made on these data using the Weibull maximum likelihood accelerated testing methology described in Reference 4.

The following stress-rupture data sets are analyzed here:

- o Kevlar 49/epoxy strands at room temperature (UV light present)
- o Kevlar 49/epoxy strands at elevated temperatures (no UV present)
- o Kevlar 49/epoxy NASA spherical vessels at room temperature (no UV present)

Included in the report is a complete collection of available relevant raw data, i.e., static strength measurements and lifetime measurements (exact, censored, and grouped), as well as summary descriptions of the materials, e.g., mean fiber weight, cross-sectional area, fiber volume content, etc.

TABLE OF CONTENTS

A. Kevlar/epoxy strand data analysis: lifetime versus stress (room temperature environment with UV light).

1. Materials

The Kevlar 49/epoxy strands used in room temperature strength and lifetime tests are described in Table 1. Added detail and descriptions of the testing equipment may be found in References 5 and 7.

Table 1. Summary of material characteristics.
(Room temperature strands)

construction: filament-wound from single end (380 denier, approximately 267 filaments) pre-production Kevlar 49 fiber without finish

epoxy matrix

- o Union Carbide ERL 2258/ZZL 0820
- o weight ratio 100/29
- o vacuum impregnation of fiber with epoxy

fiber specific gravity:	1.45
mean fiber weight:	0.0409 g/m (based on 5 specimens)
mean cross-sectional area:	4.364×10^{-5} in^2 (based on 5 specimens)
curing:	3 h at 93°C, 2 h at 163°C
mean fiber volume content:	71.5% (based on 35 specimens)

length of specimens for lifetime and strength tests: 10 in

2. Data

A strength test was performed on 53 specimens resulting in an estimated ultimate tensile strength of 505.7 ksi, with estimated coefficient of variation 2.468%. The complete set of breaking loads is presented in Appendix A. Stress-rupture (lifetime) tests were subsequently carried out at room temperature in the presence of UV light over an eight year period on a total of 553 strands at seven stress levels ranging from 90% down to 50% UTS. The data are summarized in Table 2.

Table 2. Room temperature strand data summary.

stress, ksi	(%UTS)	#strands	#exact failure times	#grouped failure times	#censored failure times	marginal ML est. $\hat{a}$	$\hat{b}$
455.2	(90.0)	101	99	2	0	1.09	-.0159
440.0	(87.0)	100	100	0	0	1.04	1.37
424.8	(84.0)	103	103	0	0	1.09	3.09
404.6	(80.0)	100	100	0	0	.923	5.37
354.0	(70.0)	49	47	2	0	.496	9.20
303.5	(60.0)	50	41	9	0	.292	10.7
252.9	(50.0)	50	4	0	46	.458	12.3

A marginal ML estimate for a given stress level is an estimate whose computation is based solely on the data from that stress level. An exact failure time is a lifetime recorded as a point, e.g. 1456.3 hours. A grouped failure time is a lifetime recorded as an interval, e.g. [1450, 1474] hours. Such is necessitated when the precise failure time of a specimen is not recorded exactly, although the time is known, as in a timer breakdown, to be a number between two specified limits. If a non-failed specimen is

removed from test independently of its condition, say because the strand slips from its clamp, or an earthquake destroys the experiment, or the decision is made to terminate all testing, the removal time, called censoring time, is recorded.

In the next section the collection of exact, grouped, and censored failure times with accompanying stresses is analyzed statistically to characterize the lifetime distribution of strands as a function of stress.

3. Statistical Analysis of the Data

The seven Weibull probability paper plots of the lifetime data partitioned according to the individual stress levels support the basic assumptions made here

(i) that the lifetime distribution for a given stress level σ is two-parameter Weibull, and

(ii) that the Weibull parameters, α (shape) and β (scale, characteristic life), depend on σ.

The statistical methodology introduced and used in the analysis of S-glass/ epoxy strand stress-rupture data (Reference 2) is accordingly appropriate also in the Kevlar setting. Thus, the lifetime T_σ of a Kevlar 49/epoxy strand under constant stress σ is modeled by the probability density function,

$$f_{T_\sigma}(t) = [\alpha(\sigma)/\beta(\sigma)][t/\beta(\sigma)]^{\alpha(\sigma)-1}\exp\{-[t/\beta(\sigma)]^{\alpha(\sigma)}\}, \ t > 0 \quad .$$

Subsequent computations are simplified by consideration of $Y_\sigma = \ell n\ T_\sigma$, whose probability distribution is that of $a(\sigma)W + b(\sigma)$, where $a(\sigma) = 1/\alpha(\sigma)$, $b(\sigma) = \ell n\ \beta(\sigma)$, and W has the extreme value density,

$$f_W(w) = \exp(w - e^w) \; , \; -\infty < w < \infty \; .$$

The effect of the transformation $Y_\sigma = \ln T_\sigma$ is a reparametrization of $\alpha(\sigma)$ and $\beta(\sigma)$ to $a(\sigma)$ and $b(\sigma)$ which behave as extreme value scale and location parameters, respectively.

Examination of marginal maximum likelihood (ML) estimates of the parameters (a, b) for the seven individual stress levels (see Table 2) suggests that the Weibull parameter to stress dependency (ii) can be modeled adequately by the polynomials

$$a(\sigma) = \theta_1 + \theta_2\sigma + \theta_3\sigma^2 \; , \text{ and}$$

$$b(\sigma) = \theta_4 + \theta_5\sigma + \theta_6\sigma^2 + \theta_7\sigma^3 \; .$$

Properties of the lifetime distribution for a given stress σ are thereby expressible in terms of the polynomial coefficients $\theta_1, \ldots, \theta_7$. Moreover, estimates of lifetime distribution characteristics of interest may be generated from estimates of $\theta_1, \ldots, \theta_7$.

Maximum likelihood estimates of the coefficients $\theta_1, \ldots, \theta_7$ were computed from the aggregated collection of 553 exact, grouped, and censored failure times recorded in terms of hours for the seven stress levels measured in ksi. The results obtained are

$$(1) \quad (\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\theta}_5, \hat{\theta}_6, \hat{\theta}_7) = (1.22750, -9.68792 \times 10^{-3}, 2.16947 \times 10^{-5}, \; 3.23296 \times 10^{1}, -2.18769 \times 10^{-1}, \; 8.11080 \times 10^{-4}, -1.07399 \times 10^{-6}).$$

Accordingly, the ML estimates of the functions $a(\sigma)$ and $b(\sigma)$ are

(2) $\hat{a}(\sigma) = \hat{\theta}_1 + \hat{\theta}_2\sigma + \hat{\theta}_3\sigma^2$ and $\hat{b}(\sigma) = \hat{\theta}_4 + \hat{\theta}_5\sigma + \hat{\theta}_6\sigma^2 + \hat{\theta}_7\sigma^3$,

and, equivalently, the ML estimates of the dependencies $\alpha(\sigma)$ and $\beta(\sigma)$ are $\alpha(\sigma) = 1/\hat{a}(\sigma)$ and $\hat{\beta}(\sigma) = \exp[\hat{b}(\sigma)]$. Of particular interest are the quantities $t_p(\sigma)$ and $R_t(\sigma)$, respectively the p^{th} quantile and the reliability at time t of the lifetime distribution for stress σ; $t_p(\sigma)$ is the time corresponding to a failure probability of p (a strand at stress σ will fail before time $t_p(\sigma)$ with probability p), and $R_t(\sigma)$ is the probability of surviving past time t (the lifetime of a strand at stress σ will exceed t with probability $R_t(\sigma)$. It is seen that $t_p(\sigma) = \exp[y_p(\sigma)]$, where $y_p(\sigma) = a(\sigma)\, w_p + b(\sigma)$, and $w_p = \ell n(-\ell n(1-p))$. Also $R_t(\sigma) = \exp\{-\exp[(\ell n\ t - b(\sigma))/a(\sigma)]\}$. The ML estimates of $t_p(\sigma)$ and $R_t(\sigma)$ are therefore

(3) $\hat{t}_p(\sigma) = \exp[\hat{y}_p(\sigma)]$, where $\hat{y}_p(\sigma) = \hat{a}(\sigma)\, w_p + \hat{b}(\sigma)$, and

(4) $\hat{R}_t(\sigma) = \exp\{-\exp[(\ell n\ t - \hat{b}(\sigma))/\hat{a}(\sigma)]\}$.

The estimated standard errors (estimated standard deviations) of all ML estimates given here are computable from results presented in Appendix A.

The maximum likelihood estimates of selected quantiles and reliabilities as functions of stress are displayed in the accompanying Figures 1 through 5. In Figure 1, the data are presented in raw fashion as sample quantiles. At each of the seven experimental stress levels, base 10 logarithms of the standard nonparametric estimates of selected quantiles are plotted. For a given quantile probability p and experimental stress level σ, this so-called Kaplan-Meier estimate is essentially the time corresponding to a failure proportion of p in the sample of failure times at

level σ. For the selected quantile probabilities p = .02, .04, .05, .10, .30, .50, and .90, these estimates, where computable, are connected by line segments. The result is a crude depiction of the relationship between stress and failure time. In Figure 2, base 10 logarithms of the maximum likelihood estimates of the same selected quantiles are plotted against stress (i.e., log $\hat{t}_p(\sigma)$ vs. σ). The general agreement of the nonparametric and ML plots is shown in Figure 3, where the competing estimates are superimposed for the quantile probabilities .02, .05,, .10, and .50.

Figures 4 and 5 give plots of estimated quantiles and reliabilities for situations of potential engineering use. In Figure 4, base 10 logarithms of the ML estimates $\hat{t}_p(\sigma)$ are plotted for the quantile probabilities $p = 10^{-6}$, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, and .50 in the range 100 to 450 ksi (19.8% to 89.0% UTS). In Figure 5, base 10 logarithms of the ML estimated failure probabilities, $1 - \hat{R}_t(\sigma)$, are plotted for the times 1, 3, 5, 10, 15, 20, 25, and 30 years in the range 125 to 275 ksi (24.7% to 54.4% UTS). To illustrate the use of these figures, consider estimating the p = .01-quantile (1st percentile) of the distribution of lifetimes of strands subjected to constant stress σ = 200 ksi.* From Figure 4, $\log \hat{t}_{.01}(200) \doteq 5.1$ so that $\hat{t}_{.01}(200) \doteq 10^{5.1} \doteq 126{,}000$ hours $\doteq$ 14.4 years. For greater accuracy, the ML estimate can be computed exactly from the equations (1), (2), and (3). Here $\hat{y}_{.01}(200) = 11.7016$ so that $\hat{t}_{.01}(200) = \exp(11.7016) = 120{,}765$ hours = 13.8 years. From results presented in Appendix A, uncertainty

* The inference applies only to the population of strands equivalent in fabrication to those tested and subject to an environment equivalent to that controlled in the experiment.

in the statistical estimation procedure can be quantified to give an approximate 90% confidence interval for $t_{.01}(200)$ of [6.82, 27.8] years. Now consider estimating the reliability at 10 years for strands at stress σ = 200 ksi.* From Figure 5, $\log(1 - \hat{R}_{10\ yr}(200)) \doteq -2.95$, which implies $\hat{R}_{10\ yr}(200) \doteq .9989$. The exact value, computed from equations (1), (2), and (4), is .9987. The corresponding approximate 90% lower confidence bound as described in Appendix A is computed to be .9926.

4. Discussion

The lifetime distribution of Kevlar 49/epoxy strands as a function of stress has been estimated by maximum likelihood methods. Formulas (1) - (4) and Figures 1, 4, and 5 provide ML estimates of various quantiles and reliabilities for a large range of stress levels. Standard errors of all ML estimates are computable from results given in Appendix A. The usefulness and applicability of these statistical inferences, however, are limited by the following sources of uncertainty.

(a) Sampling. Because lifetimes recorded at the given experimental stress levels obey probability distributions, the actual data aggregated will be diverse and, especially for small sample sizes, not necessarily representative of the underlying populations. The degree of uncertainty attributable to sampling variation for the ML estimates generated, although quantified by the appropriate estimated standard errors, can be unsatisfactorily great (i) in cases of extrapolation in stress level far beyond the data, and (ii) in estimation of extreme tail distribution parameters, e.g., $t_{10^{-3}}$ or $t_{10^{-6}}$.

(b) Weibull distribution assumption. The Weibull model is an ideal that is only approximately realized in experimentation. To the degree the Weibull assumption is violated, ML estimates are biased away from the unknown parameters being estimated. Such bias is more substantial for extreme tail parameters, e.g. $t_{10^{-3}}$, or $t_{10^{-6}}$, than for the median, $t_{.5}$. Standard errors of all ML estimates also are erroneous to the extent the Weibull assumption is violated.

(c) Assumed relationship of Weibull parameters to stress. The assumed stress models which specify the functional relationships $a(\sigma)$ and $b(\sigma)$ have a crucial bearing on all ML estimates. The selection of quadratic polynomial and cubic polynomial functions, respectively, for the LLNL data provided good agreement of ML estimates with standard nonparametric estimates (Figure 3) and the highest maximized likelihood among all functional relationships considered. In short, this selection fit the observed data quite well. Unfortunately, reasonableness within the region of the experimental stress levels does not guarantee reasonableness at extrapolated stress levels far beyond this region. For example, two sets of assumed functional dependencies, say $(a(\sigma), b(\sigma))$ and $(a^*(\sigma), b^*(\sigma))$, may give rise to respective ML estimates of parameters which within the experimental stress region are nearly coincidental. Thus $\hat{t}_{.01}(\sigma) \doteq \hat{t}^*_{.01}(\sigma)$ for all σ between 250 and 450 ksi. However, it may well be the case that $\hat{t}_{.01}(100)$ is vastly different from $\hat{t}^*_{.01}(100)$, and the true value, $t_{.01}(100)$, may be far away from either estimate. Misspecification of the functions $a(\sigma)$ and $b(\sigma)$, like violation of the Weibull distribution assumption (b), introduces bias into the ML estimates and their standard errors. This bias may be arbitrarily great at extreme stress levels.

(d) Extension to other populations. The statistical inferences based upon the given data set are strictly relevant to a hypothetical population of strands whose characterization in terms of fabrication and controlled environment is equivalent to that of the sampled strands. Application of the results to strands made from a different process or epoxy formulation, or subjected to environments with different temperature, humidity, or light conditions, introduces elements of uncertainty which at this point are not understood well enough to be quantified.

B. Kevlar 49/epoxy strand data analysis: lifetime versus stress and temperature (environment without UV light).

1. Materials

The Kevlar 49/epoxy strands used in elevated temperature lifetime tests are described in Table 3. Added detail and descriptions of the testing equipment may be found in References 6 and 7.

2. Data

Kevlar 49/epoxy strands of reportedly the same batch as in part A were subjected to stress-rupture tests at various elevated temperatures and selected stresses. No light was present during testing. The combined effect of stress and temperature on lifetime can be estimated from a statistical analysis of the observed failure times. Since the experimental elevated temperatures fell within the range 100-120°C, it was decided for interpolative purposes to include in the analysis those portions of the room temperature data of part A

Table 3. Summary of material characteristics. (Elevated temperature strands)

construction: filament-wound from single end (380 denier, approximately 267 filaments) pre-production Kevlar 49 fiber without finish

epoxy matrix

- o Union Carbide ERL 2258/ZZL 0820
- o weight ratio 100/29
- o vacuum impregation of fiber with epoxy

fiber specific gravity:	1.45
mean fiber weight:	0.0432 gm/m (based on 10 specimens)
mean cross-sectional area:	4.619×10^{-5} in^2 (based on 10 specimens)
curing:	3 h at 100°C, 2 h at 170°C
mean fiber volume content:	67.8% (based on 40 specimens)

length of specimens for lifetime and strength tests: 225 mm (8.9 in)

which were felt to be generally unaffected by the presence of UV light, namely the four highest stresses. (At the lowest of these stresses (80% UTS), all failure times were less than 1000 hours; at the highest stress excluded (70% UTS), failure times were as high as two years.) Although the composite materials under both elevated and room temperature conditions had essentially

identical descriptions (in terms of epoxy, denier, curing, fiber volume, etc.), measured cross sectional areas and ultimate tensile strengths differed slightly. Accordingly for the sake of compatibility of the two data sets, nominal experimental stress levels were given minor adjustments to achieve common proportionality to actual applied loads, and an overall UTS figure of 500 ksi was adopted. The combined data, consisting of 1190 specimen lifetimes at a total of 17 stress/temperature settings are summarized in Table 4.

3. Statistical Analysis of the Data

Weibull probability paper plots of failure time data at individual stress/temperature settings suggest that strand lifetimes may be modeled well as two-parameter Weibull random variables whose parameters α and β depend on both stress σ and temperature τ. As in the analysis of part A, use of a logarithmic transformation achieves the parametrization $a(\sigma,\tau) = 1/\alpha(\sigma,\tau)$ and $b(\sigma,\tau) = \ell n\, \beta(\sigma,\tau)$. The stress/temperature influence on a and b, by examination of maximum likelihood estimates of a and b for the 17 individual experimental settings (see Table 4), is seen to be modeled adequately as

$$a(\sigma, \tau) = \theta_1 + \theta_2\sigma + \theta_3\tau^{-1} \quad , \text{ and}$$

$$b(\sigma, \tau) = \theta_4 + \theta_5\sigma + \theta_6\sigma^2 + \theta_7\tau^{-1} + \theta_8\tau^{-2} + \theta_9\sigma\tau^{-1} \quad ,$$

where σ is measured in ksi units, and τ in degrees Kelvin (273 + degrees Centigrade). The interaction term, $\theta_9\sigma\tau^{-1}$, allows the incremental stress effect on lifetime to be different at different temperature levels (and also allows the incremental temperature effect on lifetime to be different at

Table 4. Combined elevated temperature/room temperature data summary.

stress, ksi	(%UTS)	temperature °C	#strands	#exact failure times	#grouped failure times	marginal $\hat{a}$	ML est. $\hat{b}$
436.7	(87.3)	120	48	48	0	.670	-3.27
436.7	(87.3)	110	39	39	0	.476	-2.26
436.7	(87.3)	100	46	46	0	.482	-1.77
410.7	(82.1)	120	46	46	0	.568	-1.71
410.7	(82.1)	110	78	78	0	.477	-1.25
410.7	(82.1)	100	53	53	0	.738	-.280
373.9	(74.8)	120	80	80	0	.607	.364
373.9	(74.8)	110	76	76	0	.754	.757
373.9	(74.8)	100	76	76	0	.915	1.57
343.2	(68.6)	120	73	73	0	.533	2.08
343.2	(68.6)	110	54	54	0	.592	3.14
343.2	(68.6)	100	58	58	0	.657	4.00
288.1	(57.6)	110	59	59	0	.446	5.65
447.6	(89.5)	25	101	99	2	1.09	-.0159
432.7	(86.5)	25	100	100	0	1.04	1.37
417.8	(83.6)	25	103	103	0	1.09	3.09
397.9	(79.6)	25	100	100	0	.923	5.37

different stress levels). The model for the transformed shape parameter, a, is simpler here than in the previous analysis A, a possible consequence of the absence of UV light.

The maximum likelihood estimates of the coefficients $\theta_1, \ldots, \theta_9$, computed (in a manner analogous to that of analysis A) from the collection of 1090 failure times aggregated from the 17 stress/temperature levels, are the following:

$$(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\theta}_5, \hat{\theta}_6, \hat{\theta}_7, \hat{\theta}_8, \hat{\theta}_9) = (-7.94341 \times 10^{-1}, 2.17931 \times 10^{-4},$$

$$5.23962 \times 10^{2}, -1.02439 \times 10^{2}, 1.29433 \times 10^{-1}, -3.47076 \times 10^{-5},$$

$$5.70114 \times 10^{4}, -4.21914 \times 10^{6}, -6.15226 \times 10^{1}) \quad .$$

The ML estimates of the functions a, b, α, and β are therefore $\hat{a}(\sigma,\tau) = \hat{\theta}_1 + \hat{\theta}_2\sigma + \hat{\theta}_3\tau^{-1}$, $\hat{b}(\sigma,\tau) = \hat{\theta}_4 + \hat{\theta}_5\sigma + \hat{\theta}_6\sigma^2 + \hat{\theta}_7\tau^{-1} + \hat{\theta}_8\tau^{-2} + \hat{\theta}_9\sigma\tau^{-1}$, $\hat{\alpha}(\sigma,\tau) = 1/\hat{a}(\sigma,\tau)$, and $\hat{\beta}(\sigma,\tau) = \exp[\hat{b}(\sigma,\tau)]$. Similarly, ML estimates of the lifetime parameters $t_p(\sigma,\tau)$ and $R_t(\sigma,\tau)$ are $\hat{t}_p(\sigma,\tau) = \exp[\hat{y}_p(\sigma,\tau)]$, where $\hat{y}_p(\sigma,\tau) = \hat{a}(\sigma,\tau)\ w_p + \hat{b}(\sigma,\tau)$, and $\hat{R}_t(\sigma,\tau) = \exp\{-\exp[(\ln t - \hat{b}(\sigma,\tau))/\hat{a}(\sigma,\tau)]\}$. Estimated standard errors of these ML estimates may be computed from results given in Appendix B.

Plots of selected estimated lifetime quantiles are presented in Figures 6 through 13. In Figure 6, the ML estimates $\hat{t}_p(\sigma,\tau)$ are displayed for the quantile probability p = .5 (representing the median), the temperatures 65°C, 45°C, 25°C, and the stress range 200 to 450 ksi. A similar plot for the quantile probability $p = 10^{-3}$ is presented in Figure 7. In Figures 8 and 9, the estimates $\hat{t}_p(\sigma,\tau)$ are shown for the quantile probabilities p = .5 and $p = 10^{-3}$, respectively, seven selected stress levels, and the temperature

range 20° to 70°C. In Figures 10 and 11, the temperature is held fixed, at 25° and 45°C, respectively, and the estimates $\hat{t}_p(\sigma,\tau)$ are displayed for selected quantiles and a stress range. Finally, in Figures 12 and 13, the stress level is held fixed, at 100 ksi and 200 ksi, respectively, and the estimates $\hat{t}_p(\sigma,\tau)$ are plotted for selected quantiles and a temperature range.

4. Discussion

The sources of uncertainty (a) through (d) given in part A, with the obvious extension in (c) from stress σ to (stress, temperature) = (σ,τ), are valid in the present context as well. In addition, there are uncertainties involving (e) the unquantified (assumed negligible) UV affect at the four room temperature stress levels; (f) possible (assumed nonexistent) physical differences between the strands used for room temperature testing and the strands used for elevated temperature testing; and (g) the unquantified and unmodeled effect of a higher relative humidity at room temperature than at elevated temperatures.

C. NASA Kevlar 49/epoxy spherical pressure vessel data analysis: lifetime versus pressure (room temperature environment without UV light).

1. Materials

The Kevlar 49/epoxy vessels used in room temperature strength and lifetime tests are described in Table 5. Added detail and descriptions of the testing equipment may be found in References 3, 5, 7, and 8.

Table 5. Summary of material characteristics. (Room temperature vessels)

material system:	Kevlar 49 (380 denier, approximately 267 filaments) wound in Dow DER 332/Jefferson Jeffamine T-403 with weight ratio 100/44.7
vessel geometry:	112 mm interior diameter spherical aluminum liner, 1 mm thick, overwrapped with 1.1 mm composite wall, wound in a delta axisymmetric pattern.

2. Data

The NASA pressure vessels were separated into five stress environments for stress-rupture testing at room temperature without light. Initial burst tests resulted in an estimated static strength of 5.0 ksi. The complete set of burst loads is presented in Appendix C. Each vessel was wound from one of eight different Kevlar spools of yarn. It was subsequently discovered (Reference 3) that one of the spools (spool #7) had physical properties distinctively different from the others (in terms of filament number and diameter, sodium hydroxide content, etc.). It was appropriate, then, to remove from consideration all data pertaining to specimens from this spool. The statistical analysis here is based on the exact, grouped, and censored failure times of the 139 vessels wound from the remaining seven spools, numbered 1 through 7 for convenience. (Hence spool #7 in this report refers to the original spool #8.) The stress-rupture data are summarized in Table 6. Spool identity is included in the classification due, as noted in the next section, to a significant spool effect.

3. Statistical Analysis of the Data

Again the basic Weibull lifetime distribution asumption is supported by Weibull probability paper plots. An initial investigation of the data ignored possible spool effects and treated all times at a given stess level as a sample from a single Weibull distribution. From the marginal ML estimates, $(\hat{a}, \hat{b})$ = (1.75, 5.51), (1.14, 6.73), (.952, 9.01), and (.935, 11.2) for the respective pressure levels 4.28, 3.97, 3.68, and 3.38 ksi, the dependencies $a(\sigma) = 1/\alpha(\sigma)$ and $b(\sigma) = \ell n\ \beta(\sigma)$ appeared to be modeled adequately by

$$(5) \qquad a(\sigma) = \theta_1 + \theta_2 e^{2\sigma} \quad \text{and} \quad b(\sigma) = \theta_3 + \theta_4\sigma^2 + \theta_5\sigma^3 \ .$$

Subsequently, in response to work by Gerstle (Reference 3) who drew attention to spool-to-spool variability, it was decided to incorporate spool effects into the dependency models. This was done by allowing the transformed scale parameter b to depend upon spool identity. The model ultimately selected was

$$(6) \qquad a(\sigma) = \theta_1 + \theta_2 e^{2\sigma} \text{ and } b(\sigma) = \theta_3\delta_1 + \theta_4\delta_2 + \theta_5\delta_3 + \theta_6\delta_4 + \theta_7\delta_5 + \theta_8\delta_6 + \theta_9 + \theta_{10}\,\sigma \ ,$$

where the δ_i are indicators of spool identity:

$$\delta_i = \begin{cases} 1 & \text{if the vessel is wound from spool } i \\ 0 & \text{if the vessel is wound from a spool other than spool } i \text{ or spool } 7 \\ -1 & \text{if the vessel is wound from spool } 7, \end{cases}$$

$i = 1, \ldots, 6$. The effect of spool i for $1 \leq i \leq 6$ is θ_{i+2}, the effect of spool 7 is $-(\theta_3 + \ldots + \theta_8)$, and the sum of all spool effects is zero. Thus, spool-to-spool variation is parametrized in terms of relative additive factors in transformed scale. If θ_5 equaled 0.60, say, then vessels wound from spool 3 would have lifetimes with scale parameter (characteristic life)

Table 6. Room temperature vessel data summary.

stress, ksi	(% UTS)	spool#	#vessels	#exact failure times	#grouped failure times	#censored failure times
4.28	(85.6)	1	4	4	0	0
4.28		2	8	8	0	0
4.28		3	4	4	0	0
4.28		4	5	5	0	0
4.28		5	4	4	0	0
4.28		6	3	3	0	0
4.28		7	3	3	0	0
3.97	(79.4)	1	4	4	0	0
3.97		2	7	7	0	0
3.97		3	4	4	0	0
3.97		4	4	4	0	0
3.97		5	1	1	0	0
3.97		6	2	2	0	0
3.97		7	2	2	0	0
3.68	(73.6)	1	3	3	0	0
3.68		2	6	6	0	0
3.68		3	2	2	0	0
3.68		4	2	2	0	0
3.68		5	1	1	0	0
3.68		6	3	3	0	0
3.68		7	6	5	1	0
3.38	(67.6)	1	4	0	0	4
3.38		2	1	1	0	0
3.38		3	2	1	0	1
3.38		4	5	0	0	5
3.38		5	3	3	0	0
3.38		6	3	3	0	0
3.38		7	3	0	0	3
2.49	(49.8)	1	5	0	0	5
2.49		2	6	0	0	6
2.49		3	3	0	0	3
2.49		4	7	0	0	7
2.49		5	8	0	0	8
2.49		6	7	0	0	7
2.49		7	4	0	0	4

$e^{0.60}$ = 1.82 times as great as the geometric mean characteristic life among the seven spools. The transformed shape parameter a, on the other hand, is assumed to depend only on stress and not on spool identity.

The ML fit obtained by assuming the model (6), and applying to the failure time data the basic methodology of analyses A and B, showed a substantial and dramatic improvement, as measured by maximized log likelihood, over the analogous ML fit based upon the model (5) which does not consider any spool effects. The ML estimates of the coefficients $(\theta_1, \ldots, \theta_{10})$ are the following: $(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\theta}_5, \hat{\theta}_6, \hat{\theta}_7, \hat{\theta}_8, \hat{\theta}_9, \hat{\theta}_{10}) = (3.20037 \times 10^{-1}$, 1.51037×10^{-4}, 1.09470, -8.63305×10^{-1}, -1.58238, 1.49728, -1.48675×10^{-1}, -4.28710×10^{-1}, 3.15136×10^{1}, $-6.18686)$. The ML estimated effect of spool 7 is $-(\hat{\theta}_3 + \ldots + \hat{\theta}_8) = 4.31090 \times 10^{-1}$. The ML estimates of the functions a, b, α and β are $\hat{a}(\sigma) = \hat{\theta}_1 + \hat{\theta}_2 e^{2\sigma}$, $\hat{b}(\sigma) = \hat{\theta}_3\delta_1 + \hat{\theta}_4\delta_2 + \hat{\theta}_5\delta_3 + \hat{\theta}_6\delta_4 + \hat{\theta}_7\delta_5 + \hat{\theta}_8\delta_6 + \hat{\theta}_9 + \hat{\theta}_{10}\sigma$, $\hat{\alpha}(\sigma) = 1/\hat{a}(\sigma)$, and $\hat{\beta}(\sigma) = \exp[\hat{b}(\sigma)]$. In addition, the ML estimates of the lifetime parameters $t_p(\sigma)$ and $R_t(\sigma)$ are $\hat{t}_p(\sigma) = \exp[\hat{y}_p(\sigma)]$, where $\hat{y}_p(\sigma) = \hat{a}(\sigma)\, w_p + \hat{b}(\sigma)$, and $\hat{R}_t(\sigma) = \exp\{-\exp[(\ell n\, t - \hat{b}(\sigma))/\hat{a}(\sigma)]\}$. For inferences pertinent to the "average" or "typical" spool, i.e., the conceptual spool corresponding to a zero-valued spool effect, ML estimates of the above quantities are obtained by substituting $\delta_3 = \ldots = \delta_8 = 0$ into the expressions for b and $\hat{b}$. In this case $\hat{b}(\sigma) = \hat{\theta}_9 + \hat{\theta}_{10}\,\sigma$ estimates $b(\sigma) = \theta_9 + \theta_{10}\,\sigma$. Estimated standard errors of all ML estimates are computable from results presented in Appendix C.

The maximum likelihood estimates of selected lifetime parameters as functions of spool and stress are displayed in the accompanying Figures 14 through 17. In Figure 14 and 15, ML estimates of quantiles and reliabilities, respectively, are plotted for the "average" spool case. In Figures 16 and 17, the magnitude of spool-to-spool variation is illustrated in plots of the estimated 10^{-3}-quantiles and 5-year reliabilities, respectively, for the

"best," "average," and "worst" spools. The spool which performed "best" in the testing was spool 4, whose estimated spool effect, $\hat{\theta}_6 = 1.49728$, was the largest among the seven spools. The "worst" spool, corresponding to the smallest estimated spool effect ($\hat{\theta}_5 = -1.58238$), was spool 3. Thus the "best" estimates are based on $\delta_3 = \delta_4 = \delta_5 = \delta_7 = \delta_8 = 0$, $\delta_6 = 1$; the "worst" estimates on $\delta_3 = \delta_4 = \delta_6 = \delta_7 = \delta_8 = 0$, $\delta_5 = 1$; and the "average" estimates on $\delta_3 = \delta_4 = \delta_5 = \delta_6 = \delta_7 = \delta_8 = 0$.

4. Discussion

The sources of uncertainty (a) through (d) inroduced in part A for strands are also pertinent in the vessel context. An added concern here involves the relevance of the particular spools used in the experiment. Ideally these seven spools constitute a random sample of the conceptual population of all spools producible and suitable for construction of vessels. Estimates based on the "average" spool effect (obtained above by setting each $\delta_i = 0$, $i = 3, \ldots, 8$) then pertain to the population average spool, a conceptual spool of interest. In addition, a measure of the dispersion of spool effects within the conceptual spool population is available, namely the sample standard deviation, 1.086, of the seven ML estimated spool effects. Unfortunately, however, the seven spools appear to have been selected by convenience rather than by a random scheme. Consequently, inferences cannot properly be extended to spools beyond the seven tested, and the "average" spool results pertain merely to a conceptual spool which typifies the seven.

D. Power law fits

In the absence of sources of chemical degradation, such as UV light or humidity or elevated temperatures, it has been argued (see Reference 1) that the power law model for lifetimes holds. Thus for Kevlar 49/epoxy strands at room temperature in darkness the model

$$a(\sigma) = \theta_1 \qquad \text{and} \qquad b(\sigma) = \theta_2 + \theta_3 \ln \sigma$$

would be appropriate. Based on the failure time data corresponding to the upper four stress levels from the table of part A (for which the UV effect may be assumed negligible or slight), the ML methodology yields the estimates

$$\hat{\theta}_1 = 1.04163, \quad \hat{\theta}_2 = 2.78954 \times 10^2, \quad \text{and } \hat{\theta}_3 = -4.55850 \times 10^1 \ .$$

Figure 18 displays corresponding ML estimates of selected quantiles. The results are theoretically appropriate for strands under conditions of no chemical degradation.

The power law fit applied to the NASA pressure vessel data (motivated by experimental conditions of darkness and room temperature), does not yield ML results competitive in terms of maximized log likelihood to the fit obtained using model (6) in part C. (In fairness, the aforementioned theoretical work applies to strands but not to structures.) Nonetheless, the power law model $a(\sigma) = \theta_1$ and $b(\sigma) = \theta_2\delta_1 + \theta_3\delta_2 + \theta_4\delta_3 + \theta_5\delta_4 + \theta_6\delta_5 + \theta_7\delta_6 + \theta_8 + \theta_9 \ln \sigma$ is estimated by $(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\theta}_5, \hat{\theta}_6, \hat{\theta}_7, \hat{\theta}_8, \hat{\theta}_9) = (7.81416 \times 10^{-1}, 1.17832, -9.44661 \times 10^{-1}, -1.65797, 1.68801, -3.21649 \times 10^{-1}, -5.11371 \times 10^{-1}, 3.92902 \times 10^1, -2.34652 \times 10^1)$. The ML estimated spool 7 effect is therefore $-(\hat{\theta}_2 + \ldots + \hat{\theta}_7) = 5.69321 \times 10^{-1}$. In Figure 19, the power law ML estimates of selected quantiles are plotted in terms of stress for the "average" spool.

E. Appendix A

Table 7 gives the ordered breaking loads for 53 room temperature strand specimens.

Tables 8 through 14 give the observed lifetimes for room temperature strands at seven stress levels.

Formulas presented here allow straightforward computation of ML estimates and their estimated standard errors (i.e., square roots of estimated variances) for quantities introduced in part A.

From equations (2) and (3), it follows that $\hat{y}_p(\sigma)$ can be expressed as a third degree polynomial in σ, namely

$$\hat{y}_p(\sigma) = (\hat{\theta}_1 w_p + \hat{\theta}_4) + (\hat{\theta}_2 w_p + \hat{\theta}_5) \sigma + (\hat{\theta}_3 w_p + \hat{\theta}_6) \sigma^2 + \hat{\theta}_7 \sigma^3 .$$

The graphed versions of Figures 1, 3, and 4 involve $\log \hat{t}_p(\sigma) = (\log e)$ $\hat{y}_p(\sigma)$, where $\log e = 0.4342944819$.

Table 7. Ordered breaking loads, in pounds, of room temperature Kevlar 49/epoxy strand specimens.

Breaking load, lb.	Quantity
20.5	1
20.9	1
21.1	1
21.4	5
21.6	6
21.8	4
22.0	13
22.2	5
22.5	8
22.7	5
22.9	3
23.1	1
	53

TABLE 8.
Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 455.2 ksi
Temperature: 25 deg C.
Number of specimens: 101

1	g	27	0.24	53	0.83	79	1.52
2	g	28	0.29	54	0.85	80	1.53
3	0.02	29	0.34	55	0.90	81	1.54
4	0.02	30	0.35	56	0.92	82	1.54
5	0.02	31	0.36	57	0.95	83	1.55
6	0.03	32	0.38	58	0.99	84	1.58
7	0.03	33	0.40	59	1.00	85	1.60
8	0.04	34	0.42	60	1.01	86	1.63
9	0.05	35	0.43	61	1.02	87	1.64
10	0.06	36	0.52	62	1.03	88	1.80
11	0.07	37	0.54	63	1.05	89	1.80
12	0.07	38	0.56	64	1.10	90	1.81
13	0.08	39	0.60	65	1.10	91	2.02
14	0.09	40	0.60	66	1.11	92	2.05
15	0.09	41	0.63	67	1.15	93	2.14
16	0.10	42	0.65	68	1.18	94	2.17
17	0.10	43	0.67	69	1.20	95	2.33
18	0.11	44	0.68	70	1.29	96	3.03
19	0.11	45	0.72	71	1.31	97	3.03
20	0.12	46	0.72	72	1.33	98	3.34
21	0.13	47	0.72	73	1.34	99	4.20
22	0.18	48	0.73	74	1.40	100	4.69
23	0.19	49	0.79	75	1.43	101	7.89
24	0.20	50	0.79	76	1.45		
25	0.23	51	0.80	77	1.50		
26	0.24	52	0.80	78	1.51		

g Grouped time in the interval (0.00,0.01).

TABLE 9.
Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 440.0 ksi
Temperature: 25 deg C.
Number of specimens: 100

1	0.03	26	1.35	51	2.80	76	5.92
2	0.04	27	1.35	52	2.89	77	6.19
3	0.07	28	1.36	53	2.91	78	6.33
4	0.08	29	1.39	54	2.91	79	6.59
5	0.08	30	1.40	55	3.10	80	6.62
6	0.09	31	1.48	56	3.11	81	6.94
7	0.15	32	1.55	57	3.42	82	7.16
8	0.15	33	1.56	58	3.47	83	7.29
9	0.20	34	1.57	59	3.50	84	7.36
10	0.28	35	1.59	60	3.64	85	7.38
11	0.28	36	1.80	61	3.67	86	7.42
12	0.30	37	1.85	62	3.68	87	8.09
13	0.38	38	1.86	63	3.75	88	8.88
14	0.41	39	1.92	64	4.39	89	9.05
15	0.50	40	2.00	65	4.50	90	9.25
16	0.51	41	2.08	66	4.53	91	9.26
17	0.57	42	2.27	67	4.53	92	9.47
18	0.70	43	2.38	68	4.65	93	10.50
19	0.72	44	2.46	69	4.65	94	10.57
20	0.82	45	2.49	70	4.69	95	11.25
21	0.84	46	2.61	71	4.70	96	13.49
22	1.05	47	2.61	72	4.75	97	13.78
23	1.13	48	2.62	73	4.84	98	16.15
24	1.19	49	2.74	74	5.01	99	16.59
25	1.32	50	2.79	75	5.71	100	18.16

TABLE 10.
Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 424.8 ksi
Temperature: 25 deg C.
Number of specimens: 103

1	0.25	27	7.32	53	12.94	79	34.51
2	0.31	28	7.45	54	16.42	80	34.74
3	0.44	29	7.92	55	16.47	81	35.63
4	0.45	30	7.99	56	17.68	82	36.84
5	0.57	31	8.22	57	17.79	83	36.97
6	0.65	32	8.35	58	18.45	84	39.17
7	0.73	33	8.38	59	18.95	85	41.54
8	0.89	34	8.39	60	19.07	86	42.32
9	1.06	35	8.45	61	19.40	87	42.53
10	1.22	36	8.53	62	19.62	88	43.61
11	1.37	37	8.55	63	19.86	89	48.54
12	1.83	38	8.64	64	20.76	90	49.02
13	1.96	39	8.68	65	21.38	91	55.20
14	2.15	40	8.92	66	23.03	92	55.99
15	2.40	41	8.93	67	23.10	93	61.37
16	2.51	42	9.45	68	23.83	94	63.17
17	2.77	43	9.57	69	24.46	95	66.48
18	4.05	44	9.80	70	24.81	96	67.06
19	4.07	45	9.83	71	25.15	97	74.01
20	4.34	46	10.60	72	25.18	98	74.61
21	4.98	47	10.82	73	26.96	99	76.46
22	5.86	48	10.83	74	27.53	100	84.26
23	5.90	49	11.03	75	27.86	101	89.87
24	6.18	50	11.12	76	29.89	102	97.37
25	6.30	51	11.13	77	32.55	103	119.09
26	7.14	52	12.52	78	33.95		

TABLE 11.

Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 404.6 ksi

Temperature: 25 deg C.

Number of specimens: 100

1	1.8	26	84.2	51	152.2	76	285.9
2	3.1	27	87.1	52	152.8	77	292.6
3	4.2	28	87.3	53	157.7	78	295.1
4	6.0	29	93.2	54	160.0	79	301.1
5	7.5	30	103.4	55	163.6	80	304.3
6	8.2	31	104.6	56	166.9	81	316.8
7	8.5	32	105.5	57	170.5	82	329.8
8	10.3	33	108.8	58	174.9	83	334.1
9	10.6	34	112.6	59	177.7	84	346.2
10	24.2	35	116.8	60	179.2	85	351.2
11	29.6	36	118.0	61	183.6	86	353.3
12	31.7	37	122.0	62	183.8	87	369.3
13	41.9	38	123.5	63	194.3	88	372.3
14	44.1	39	124.4	64	195.1	89	381.3
15	49.5	40	125.4	65	195.3	90	393.5
16	50.1	41	129.5	66	202.6	91	451.3
17	59.7	42	130.4	67	220.2	92	461.5
18	61.7	43	131.6	68	221.3	93	574.2
19	64.4	44	132.8	69	227.2	94	653.3
20	69.7	45	133.8	70	251.0	95	663.0
21	70.0	46	137.0	71	266.5	96	669.8
22	77.8	47	140.2	72	267.9	97	739.7
23	80.5	48	140.9	73	269.2	98	759.6
24	82.3	49	148.5	74	270.4	99	894.7
25	83.5	50	149.2	75	272.5	100	974.9

TABLE 12.
Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 354.0 ksi
Temperature: 25 deg C.
Number of specimens: 49

1	1051.	14	5817.	27	9711.	40	12044.
2	1337.	15	5905.	28	9806.	41	13520.
3	1389.	16	5956.	29	10205.	42	13670.
4	1921.	17	6068.	30	10396.	43	14110.
5	1942.	18	6121.	31	10861.	44	14496.
6	2322.	19	6473.	32	11026.	45	15395.
7	3629.	20	7501.	33	11214.	46	16179.
8	4006.	21	7886.	34	11362.	47	17092.
9	4012.	22	8108.	35	11604.	48	g
10	4063.	23	8546.	36	11608.	49	g
11	4921.	24	8666.	37	11745.		
12	5445.	25	8831.	38	11762.		
13	5620.	26	9106.	39	11895.		

g Grouped time in the interval (17408.,17576.).

TABLE 13.
Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 303.5 ksi
Temperature: 25 deg C.
Number of specimens: 50

1	13872.	14	29832.	27	39240.	40	49440.
2	18024.	15	31224.	28	39576.	41	51192.
3	19008.	16	31752.	29	39744.	42	g1
4	21960.	17	32232.	30	39744.	43	g1
5	22872.	18	32976.	31	41592.	44	g2
6	25008.	19	35544.	32	41760.	45	g2
7	25848.	20	35760.	33	41760.	46	g2
8	27216.	21	35928.	34	42600.	47	g2
9	27744.	22	36528.	35	42600.	48	g2
10	27840.	23	36720.	36	42960.	49	g2
11	28512.	24	38592.	37	43176.	50	g2
12	28896.	25	38592.	38	44664.		
13	29832.	26	39072.	39	49176.		

g1 Grouped time in the interval (49080.,66408.).
g2 Grouped time in the interval (54744.,72072.).

TABLE 14.
Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 252.9 ksi
Temperature: 25 deg C.
Number of specimens: 50

1	31344.	2	32376.	3	58056.	4	66024.

Censored Times and Quantities

10000.	1
14376.	1
66408.	9
72072.	35

From large sample statistical theory, the joint probability distribution of the ML estimates $(\hat{\theta}_1, \ldots, \hat{\theta}_7)$ is approximately 7-dimensional multivariate normal with mean vector $(\theta_1, \ldots, \theta_7)$ and covariance matrix Σ estimated by $\hat{\Sigma} = (s_{ij})$, i, j = 1, ..., 7,

$$= \begin{bmatrix} 4.7576 \times 10^{-1} & -2.8951 \times 10^{-3} & 4.2251 \times 10^{-6} & 2.1477 \times 10^{0} & -1.9355 \times 10^{-2} & 5.7394 \times 10^{-5} & -5.5780 \times 10^{-8} \\ -2.8951 \times 10^{-3} & 1.7743 \times 10^{-5} & -2.6046 \times 10^{-8} & -1.2800 \times 10^{-2} & 1.1607 \times 10^{-4} & -3.4605 \times 10^{-7} & 3.3785 \times 10^{-10} \\ 4.2251 \times 10^{-6} & -2.6046 \times 10^{-8} & 3.8449 \times 10^{-11} & 1.8434 \times 10^{-5} & -1.6787 \times 10^{-7} & 5.0227 \times 10^{-10} & -4.9186 \times 10^{-13} \\ 2.1477 \times 10^{0} & -1.2800 \times 10^{-2} & 1.8434 \times 10^{-5} & 4.2525 \times 10^{1} & -3.7848 \times 10^{-1} & 1.1021 \times 10^{-3} & -1.0494 \times 10^{-6} \\ -1.9355 \times 10^{-2} & 1.1607 \times 10^{-4} & -1.6787 \times 10^{-7} & -3.7848 \times 10^{-1} & 3.3820 \times 10^{-3} & -9.8833 \times 10^{-6} & 9.4416 \times 10^{-9} \\ 5.7394 \times 10^{-5} & -3.4605 \times 10^{-7} & 5.0227 \times 10^{-10} & 1.1021 \times 10^{-3} & -9.8833 \times 10^{-6} & 2.8980 \times 10^{-8} & -2.7770 \times 10^{-11} \\ -5.5780 \times 10^{-8} & 3.3785 \times 10^{-10} & -4.9186 \times 10^{-13} & -1.0494 \times 10^{-6} & 9.4416 \times 10^{-9} & -2.7770 \times 10^{-11} & -2.6685 \times 10^{-14} \end{bmatrix}$$

The corresponding estimates of the variances of the estimators $\hat{a}(\sigma)$ and $\hat{b}(\sigma)$ are expressible as polynomials in σ (where $(c_1, \ldots, c_7) = (1, \sigma, \sigma^2, 1, \sigma, \sigma^2, \sigma^3)$):

$$\widehat{\text{Var}}(\hat{a}(\sigma)) = \sum_{i=1}^{3} \sum_{j=1}^{3} c_i\, c_j\, s_{ij} = s_{11} + 2\, s_{12}\, \sigma + (s_{22} + 2\, s_{13})\sigma^2 + 2\, s_{23}\, \sigma^3 + s_{33}\, \sigma^4, \text{ and}$$

$$\widehat{\text{Var}}(\hat{b}(\sigma)) = \sum_{i=4}^{7} \sum_{j=4}^{7} c_i c_j s_{ij} = s_{44} + 2\, s_{45}\, \sigma + (s_{55} + 2\, s_{46})\sigma^2 + 2(s_{47} + s_{56})\sigma^3 + (s_{66} + 2\, s_{57})\sigma^4 + 2\, s_{67}\, \sigma^5 + s_{77}\, \sigma^6 .$$

Similarly, the estimated covariance between $\hat{a}(\sigma)$ and $\hat{b}(\sigma)$ is expressible as

$$\widehat{\mathrm{Cov}}(\hat{a}(\sigma),\ \hat{b}(\sigma)) = \sum_{i=1}^{3} \sum_{j=4}^{7} c_i c_j s_{ij} = s_{14} + (s_{15} + s_{24})\sigma + (s_{16} + s_{25} + s_{34})\sigma^2$$

$$+ (s_{17} + s_{26} + s_{35})\sigma^3 + (s_{27} + s_{36})\sigma^4 + s_{37}\,\sigma^5 \quad .$$

Thus, estimates of the variances of $\hat{y}_p(\sigma)$ and $\hat{R}_t(\sigma)$ are

$$\widehat{\mathrm{Var}}(\hat{y}_p(\sigma)) = w_p^{\ 2}\ \widehat{\mathrm{Var}}(\hat{a}(\sigma)) + \widehat{\mathrm{Var}}(\hat{b}(\sigma)) + 2\ w_p\ \widehat{\mathrm{Cov}}(\hat{a}(\sigma),\ \hat{b}(\sigma)),\ \text{and}$$

$$\mathrm{Var}(\hat{R}_t(\sigma)) = \left\{\frac{1}{\hat{a}(\sigma)} \exp[(\ln t - \hat{b}(\sigma))/\hat{a}(\sigma)]\ \hat{R}_t(\sigma)\right\}^2$$

$$\{[(\ln t - \hat{b}(\sigma))/\hat{a}(\sigma)]^2\ \widehat{\mathrm{Var}}(\hat{a}(\sigma)) + \widehat{\mathrm{Var}}(\hat{b}(\sigma))$$

$$+ 2[(\ln t - \hat{b}(\sigma))/\hat{a}(\sigma)]\ \widehat{\mathrm{Cov}}\ (\hat{a}(\sigma),\ \hat{b}(\sigma))\} \quad .$$

A confidence interval for the quantile $y_p(\sigma)$ with confidence coefficient approximately $1-\alpha$ is therefore $\hat{y}_p(\sigma) \pm z_{1-\alpha/2}[\widehat{\mathrm{Var}}(\hat{y}_p(\sigma))]^{1/2}$, where z_γ denotes the γ-quantile of the standard normal distribution (e.g., $z_{.95} = 1.645$). The corresponding confidence interval for the quantile $t_p(\sigma)$ is accordingly

$$[\exp\{\hat{y}_p(\sigma) - z_{1-\alpha/2}[\widehat{\mathrm{Var}}(\hat{y}_p(\sigma))]^{1/2}\},\ \exp\{\hat{y}_p(\sigma) + z_{1-\alpha/2}[\widehat{\mathrm{Var}}(\hat{y}_p(\sigma))]^{1/2}\}].$$

Similarly, a lower confidence bound for $R_t(\sigma)$ with confidence coefficient approximately $1-\alpha$ is $\hat{R}_t(\sigma) - z_{1-\alpha}[\widehat{\mathrm{Var}}(\hat{R}_t(\sigma))]^{1/2}$.

F. Appendix B

Results analogous to those of Appendix A are presented here to provide assistance in computations of quantities introduced in part B.

Strength tests were performed on 13 strand specimens from the lot used in subsequent elevated temperature stress-rupture tests. For the 13 strands,

breaking loads were measured at room temperature. The individual values have been lost. However, their arithmetic average, 22.66 lb, has survived.

Tables 15 through 27 give the lifetime data for the 13 environments at elevated temperatures. (The lifetime data for the four environments at room temperature used in the (stress, temperature) study are given in Tables 8 through 11 of Appendix A.)

The natural logarithm of the ML estimate $\hat{t}_p(\sigma,\tau)$ is expressible as

$$\hat{y}_p(\sigma,\tau) = (\hat{\theta}_1 w_p + \hat{\theta}_4) + (\hat{\theta}_2 w_p + \hat{\theta}_5)\sigma + \hat{\theta}_6\sigma^2 + (\hat{\theta}_3 w_p + \hat{\theta}_7)\tau^{-1} + \hat{\theta}_8\tau^{-2} + \hat{\theta}_9\sigma\tau^{-1}.$$

The joint probability distribution of the ML estimates $(\hat{\theta}_1, \ldots, \hat{\theta}_9)$ is approximately 9-dimensional multivariate normal with mean vector $(\theta_1, \ldots, \theta_9)$ and corvariance matrix Σ estimated by $\hat{\Sigma} = (s_{ij})$, $i,j = 1, \ldots, 9$,

$$= \begin{bmatrix}
4.0994 \times 10^{-2} & -6.1100 \times 10^{-5} & -6.3192 \times 10^{0} & -2.0055 \times 10^{-2} & -2.6208 \times 10^{-4} & 6.4099 \times 10^{-7} & 4.1896 \times 10^{1} & -1.4029 \times 10^{3} & -7.4974 \times 10^{-2} \\
-6.1100 \times 10^{-5} & 2.7543 \times 10^{-7} & -1.6482 \times 10^{-2} & -5.8044 \times 10^{-5} & 1.3207 \times 10^{-6} & -2.4575 \times 10^{-9} & -1.2160 \times 10^{-1} & 1.0320 \times 10^{1} & 1.6345 \times 10^{-4} \\
-6.3192 \times 10^{0} & -1.6482 \times 10^{-2} & 4.6484 \times 10^{3} & 1.6165 \times 10^{1} & -9.1467 \times 10^{-2} & 1.1590 \times 10^{-4} & 1.4935 \times 10^{3} & -8.4062 \times 10^{5} & 3.7060 \times 10^{0} \\
-2.0055 \times 10^{-2} & -5.8044 \times 10^{-5} & 1.6165 \times 10^{1} & 3.3264 \times 10^{1} & -2.9126 \times 10^{-2} & 1.5201 \times 10^{-5} & -1.9016 \times 10^{4} & 2.7260 \times 10^{6} & 6.6757 \times 10^{0} \\
-2.6208 \times 10^{-4} & 1.3207 \times 10^{-6} & -9.1467 \times 10^{-2} & -2.9126 \times 10^{-2} & 1.2683 \times 10^{-4} & -7.6153 \times 10^{-8} & 3.6954 \times 10^{0} & 1.2593 \times 10^{3} & -2.6377 \times 10^{-2} \\
6.4099 \times 10^{-7} & -2.4575 \times 10^{-9} & 1.1590 \times 10^{-4} & 1.5201 \times 10^{-5} & -7.6153 \times 10^{-8} & 1.6770 \times 10^{-10} & -6.7916 \times 10^{-4} & 1.3967 \times 10^{0} & -1.8669 \times 10^{-5} \\
4.1896 \times 10^{1} & -1.2160 \times 10^{-1} & 1.4935 \times 10^{3} & -1.9016 \times 10^{4} & 3.6954 \times 10^{0} & -6.7916 \times 10^{-4} & 1.2535 \times 10^{7} & -2.0262 \times 10^{9} & -1.1950 \times 10^{3} \\
-1.4029 \times 10^{3} & 1.0320 \times 10^{1} & -8.4062 \times 10^{5} & 2.7260 \times 10^{6} & 1.2593 \times 10^{3} & 1.3967 \times 10^{0} & -2.0262 \times 10^{9} & 4.0321 \times 10^{11} & -8.7713 \times 10^{5} \\
-7.4974 \times 10^{-2} & 1.6345 \times 10^{-4} & 3.7060 \times 10^{0} & 6.6757 \times 10^{0} & -2.6377 \times 10^{-2} & -1.8669 \times 10^{-5} & -1.1950 \times 10^{3} & -8.7713 \times 10^{5} & 1.5296 \times 10^{1}
\end{bmatrix}$$

TABLE 15.
Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 436.7 ksi
Temperature: 120 deg C.
Number of specimens: 48

1	0.0068	13	0.0162	25	0.0247	37	0.0425
2	0.0072	14	0.0175	26	0.0262	38	0.0478
3	0.0080	15	0.0177	27	0.0290	39	0.0485
4	0.0095	16	0.0182	28	0.0308	40	0.0547
5	0.0118	17	0.0185	29	0.0317	41	0.0665
6	0.0122	18	0.0190	30	0.0332	42	0.0700
7	0.0130	19	0.0217	31	0.0333	43	0.0747
8	0.0132	20	0.0233	32	0.0342	44	0.0752
9	0.0150	21	0.0233	33	0.0350	45	0.0763
10	0.0152	22	0.0235	34	0.0383	46	0.0815
11	0.0152	23	0.0242	35	0.0403	47	0.0830
12	0.0162	24	0.0245	36	0.0413	48	0.1247

TABLE 16.
Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 436.7 ksi
Temperature: 110 deg C.
Number of specimens: 39

1	0.0148	11	0.0593	21	0.0885	31	0.1395
2	0.0248	12	0.0595	22	0.0900	32	0.1410
3	0.0263	13	0.0605	23	0.0910	33	0.1448
4	0.0301	14	0.0618	24	0.0928	34	0.1516
5	0.0323	15	0.0713	25	0.0933	35	0.1663
6	0.0461	16	0.0728	26	0.0963	36	0.1727
7	0.0530	17	0.0740	27	0.1113	37	0.1762
8	0.0573	18	0.0770	28	0.1240	38	0.1783
9	0.0580	19	0.0775	29	0.1330	39	0.1783
10	0.0593	20	0.0830	30	0.1347		

TABLE 17.
Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 436.7 ksi
Temperature: 100 deg C.
Number of specimens: 46

1	0.0407	13	0.0890	25	0.1517	37	0.2080
2	0.0437	14	0.0977	26	0.1567	38	0.2249
3	0.0463	15	0.1063	27	0.1612	39	0.2262
4	0.0480	16	0.1087	28	0.1613	40	0.2432
5	0.0526	17	0.1119	29	0.1662	41	0.2478
6	0.0532	18	0.1223	30	0.1750	42	0.2497
7	0.0545	19	0.1233	31	0.1876	43	0.2537
8	0.0587	20	0.1347	32	0.1912	44	0.2873
9	0.0697	21	0.1362	33	0.1932	45	0.2917
10	0.0718	22	0.1432	34	0.1950	46	0.3762
11	0.0770	23	0.1463	35	0.1963		
12	0.0837	24	0.1500	36	0.2032		

TABLE 18.
Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 410.7 ksi
Temperature: 120 deg C.
Number of specimens: 46

1	0.0102	13	0.0997	25	0.1452	37	0.2567
2	0.0200	14	0.1078	26	0.1472	38	0.2578
3	0.0200	15	0.1082	27	0.1548	39	0.2597
4	0.0612	16	0.1113	28	0.1607	40	0.2730
5	0.0650	17	0.1170	29	0.1607	41	0.2798
6	0.0738	18	0.1213	30	0.1633	42	0.3443
7	0.0749	19	0.1222	31	0.1695	43	0.3493
8	0.0752	20	0.1248	32	0.1707	44	0.3493
9	0.0800	21	0.1280	33	0.1747	45	0.3607
10	0.0862	22	0.1337	34	0.1897	46	0.3967
11	0.0879	23	0.1419	35	0.2133		
12	0.0970	24	0.1440	36	0.2162		

TABLE 19.
Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 410.7 ksi
Temperature: 110 deg C.
Number of specimens: 78

1	0.0095	21	0.1707	41	0.2487	61	0.3577
2	0.0113	22	0.1780	42	0.2513	62	0.3633
3	0.0217	23	0.1825	43	0.2630	63	0.3663
4	0.0602	24	0.1848	44	0.2635	64	0.3673
5	0.0797	25	0.1865	45	0.2650	65	0.3753
6	0.0855	26	0.1880	46	0.2658	66	0.3758
7	0.0985	27	0.1915	47	0.2735	67	0.3802
8	0.0995	28	0.1958	48	0.2762	68	0.3858
9	0.1063	29	0.2013	49	0.2828	69	0.3942
10	0.1232	30	0.2032	50	0.2928	70	0.3977
11	0.1272	31	0.2083	51	0.2947	71	0.4268
12	0.1315	32	0.2170	52	0.2957	72	0.4433
13	0.1347	33	0.2195	53	0.3008	73	0.4478
14	0.1423	34	0.2217	54	0.3068	74	0.4745
15	0.1432	35	0.2235	55	0.3087	75	0.5010
16	0.1453	36	0.2313	56	0.3193	76	0.5115
17	0.1497	37	0.2345	57	0.3252	77	0.5518
18	0.1603	38	0.2437	58	0.3267	78	0.5685
19	0.1665	39	0.2463	59	0.3278		
20	0.1682	40	0.2478	60	0.3492		

TABLE 20.
Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 410.7 ksi
Temperature: 100 deg C.
Number of specimens: 53

1	0.0392	15	0.2882	29	0.5972	43	1.1030
2	0.0815	16	0.2983	30	0.6042	44	1.1370
3	0.0875	17	0.2997	31	0.6465	45	1.1380
4	0.1233	18	0.3087	32	0.7013	46	1.1750
5	0.1342	19	0.3183	33	0.7042	47	1.2720
6	0.1787	20	0.3697	34	0.7808	48	1.3200
7	0.1833	21	0.4415	35	0.8052	49	1.4800
8	0.1880	22	0.4620	36	0.8473	50	1.8320
9	0.1982	23	0.4625	37	0.8567	51	2.0690
10	0.2533	24	0.5312	38	0.8732	52	2.0700
11	0.2697	25	0.5535	39	0.8743	53	2.1640
12	0.2697	26	0.5550	40	0.9833		
13	0.2782	27	0.5550	41	0.9907		
14	0.2783	28	0.5837	42	1.0070		

TABLE 21.
Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 373.9 ksi
Temperature: 120 deg C.
Number of specimens: 80

1	0.1427	21	0.6520	41	1.2043	61	1.5947
2	0.1782	22	0.6600	42	1.2063	62	1.6583
3	0.2633	23	0.6762	43	1.2092	63	1.6828
4	0.2967	24	0.7008	44	1.2247	64	1.6887
5	0.3900	25	0.7923	45	1.2278	65	1.8027
6	0.4217	26	0.8112	46	1.2411	66	1.8480
7	0.4278	27	0.8143	47	1.2800	67	1.9000
8	0.4318	28	0.8268	48	1.3070	68	2.1300
9	0.4375	29	0.8585	49	1.3091	69	2.1980
10	0.4390	30	0.9097	50	1.3345	70	2.2075
11	0.4542	31	0.9142	51	1.3732	71	2.2110
12	0.5313	32	0.9362	52	1.3913	72	2.3642
13	0.5330	33	0.9562	53	1.4313	73	2.4338
14	0.5353	34	0.9566	54	1.4352	74	2.6652
15	0.5397	35	0.9876	55	1.4415	75	2.8480
16	0.5538	36	0.9920	56	1.4430	76	2.9662
17	0.5748	37	0.9990	57	1.4980	77	3.1533
18	0.5942	38	1.0075	58	1.5045	78	3.3112
19	0.6197	39	1.1120	59	1.5045	79	3.7097
20	0.6323	40	1.1547	60	1.5180	80	4.2721

TABLE 22.

Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 373.9 ksi

Temperature: 110 deg C.

Number of specimens: 76

1	0.0251	20	0.9113	39	1.7460	58	2.3203
2	0.0886	21	0.9120	40	1.7630	59	2.3470
3	0.0891	22	0.9836	41	1.7746	60	2.3513
4	0.2501	23	1.0483	42	1.8275	61	2.4951
5	0.3113	24	1.0596	43	1.8375	62	2.5260
6	0.3451	25	1.0773	44	1.8503	63	2.9911
7	0.4763	26	1.1733	45	1.8808	64	3.0256
8	0.5650	27	1.2570	46	1.8878	65	3.2678
9	0.5671	28	1.2766	47	1.8881	66	3.4045
10	0.6566	29	1.2985	48	1.9316	67	3.4846
11	0.6748	30	1.3211	49	1.9558	68	3.7433
12	0.6751	31	1.3503	50	2.0048	69	3.7455
13	0.6753	32	1.3551	51	2.0408	70	3.9143
14	0.7696	33	1.4595	52	2.0903	71	4.8073
15	0.8375	34	1.4880	53	2.1093	72	5.4005
16	0.8391	35	1.5728	54	2.1330	73	5.4435
17	0.8425	36	1.5733	55	2.2100	74	5.5295
18	0.8645	37	1.7083	56	2.2460	75	6.5541
19	0.8851	38	1.7263	57	2.2878	76	9.0960

TABLE 23.

Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 373.9 ksi

Temperature: 100 deg C.

Number of specimens: 76

1	0.1050	20	1.7042	39	3.6768	58	6.6752
2	0.1367	21	1.7267	40	3.7078	59	6.7150
3	0.1516	22	1.7276	41	3.7095	60	6.8033
4	0.2833	23	1.8098	42	3.7997	61	6.8560
5	0.3013	24	1.8292	43	4.0242	62	6.8616
6	0.3882	25	2.0078	44	4.0253	63	7.2442
7	0.3902	26	2.2367	45	4.2678	64	7.5138
8	0.4083	27	2.7045	46	4.3113	65	8.0610
9	0.4378	28	2.7721	47	4.4470	66	8.3400
10	0.5113	29	2.8795	48	4.4788	67	8.4076
11	0.5863	30	2.9480	49	4.5420	68	8.4940
12	1.1243	31	2.9503	50	4.6260	69	9.0733
13	1.2750	32	3.0147	51	4.9577	70	9.4785
14	1.2900	33	3.0525	52	5.2326	71	10.2180
15	1.4228	34	3.1625	53	5.9757	72	11.2280
16	1.4651	35	3.3065	54	6.0613	73	14.7858
17	1.4882	36	3.3260	55	6.0955	74	17.2493
18	1.5178	37	3.4085	56	6.1063	75	19.8447
19	1.6163	38	3.4562	57	6.6546	76	23.0297

TABLE 24.
Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 343.2 ksi
Temperature: 120 deg C.
Number of specimens: 73

1	0.7367	20	3.9653	39	6.3163	58	9.9316
2	1.1627	21	4.2488	40	6.4513	59	10.0180
3	1.8945	22	4.3017	41	6.8320	60	10.4028
4	1.9340	23	4.3942	42	6.9447	61	10.4188
5	2.3180	24	4.6416	43	7.2595	62	10.7250
6	2.6483	25	4.7070	44	7.3183	63	10.9411
7	2.8573	26	4.8885	45	7.3313	64	11.7962
8	2.9918	27	5.1746	46	7.7587	65	12.0750
9	3.0797	28	5.4962	47	8.0393	66	12.6933
10	3.1152	29	5.5310	48	8.0693	67	13.5307
11	3.1335	30	5.5588	49	8.1928	68	13.8105
12	3.2647	31	5.6333	50	8.4166	69	14.5067
13	3.4873	32	5.7006	51	8.7558	70	15.3013
14	3.5380	33	5.8730	52	8.8398	71	16.2742
15	3.6335	34	5.8737	53	9.2497	72	18.2682
16	3.6541	35	5.9378	54	9.2563	73	19.2033
17	3.7645	36	6.1960	55	9.5418		
18	3.8196	37	6.2217	56	9.6472		
19	3.8520	38	6.2630	57	9.6902		

TABLE 25.
Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 343.2 ksi
Temperature: 110 deg C.
Number of specimens: 54

1	3.4347	15	10.9848	29	18.0317	43	30.5233
2	5.0883	16	11.1485	30	19.2267	44	30.5433
3	5.2947	17	11.4787	31	19.4867	45	30.6183
4	5.8670	18	11.4930	32	19.5367	46	30.9917
5	7.1213	19	11.6778	33	19.7300	47	31.4000
6	8.5342	20	12.6927	34	21.5833	48	35.0900
7	8.6980	21	12.9188	35	22.5367	49	37.8417
8	8.7880	22	13.4178	36	22.8000	50	39.2433
9	9.2728	23	14.3510	37	24.4950	51	40.8633
10	9.6667	24	14.8937	38	26.4000	52	45.1067
11	9.8350	25	15.0912	39	27.0783	53	60.1983
12	9.9262	26	15.1522	40	28.1317	54	63.6067
13	10.0907	27	16.6075	41	28.5900		
14	10.6818	28	17.5983	42	28.6517		

TABLE 26.

Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 343.2 ksi

Temperature: 100 deg C.

Number of specimens: 58

1	4.5145	16	26.1750	31	41.6983	46	65.8350
2	7.4662	17	26.9767	32	41.9100	47	65.9333
3	11.3375	18	27.2833	33	42.0117	48	70.9167
4	12.9017	19	27.8417	34	43.9867	49	74.5300
5	14.5092	20	27.9033	35	46.0433	50	74.7133
6	18.2700	21	28.5850	36	47.1717	51	87.0617
7	18.2867	22	29.7867	37	47.6983	52	102.2000
8	18.4283	23	33.2800	38	48.6550	53	103.5917
9	18.9767	24	35.0933	39	49.3833	54	105.4233
10	19.6067	25	35.3183	40	51.9239	55	109.3800
11	21.0517	26	36.7000	41	52.2283	56	138.4283
12	24.5867	27	36.9050	42	54.1567	57	156.8033
13	25.2250	28	39.6950	43	57.8333	58	168.6850
14	25.4050	29	41.1783	44	59.4983		
15	26.1517	30	41.4117	45	60.6517		

TABLE 27.
Ranks and Breaking Times (hr) for Kevlar 49/Epoxy Strands

Stress: 288.1 ksi
Temperature: 110 deg C.
Number of specimens: 59

1	58.	16	164.	31	240.	46	360.
2	66.	17	168.	32	252.	47	360.
3	66.	18	168.	33	257.	48	360.
4	72.	19	168.	34	266.	49	360.
5	74.	20	172.	35	275.	50	376.
6	96.	21	186.	36	278.	51	384.
7	102.	22	192.	37	285.	52	402.
8	110.	23	196.	38	292.	53	408.
9	120.	24	198.	39	327.	54	418.
10	120.	25	209.	40	336.	55	432.
11	120.	26	216.	41	336.	56	462.
12	132.	27	232.	42	337.	57	463.
13	135.	28	236.	43	338.	58	482.
14	143.	29	237.	44	338.	59	523.
15	144.	30	240.	45	357.		

The corresponding estimates of the variances of the estimators $\hat{a}(\sigma,\tau)$ and $\hat{b}(\sigma,\tau)$, and of the covariance between $\hat{a}(\sigma,\tau)$ and $\hat{b}(\sigma,\tau)$, are $\widehat{Var}(\hat{a}(\sigma,\tau)) = \sum_{i=1}^{3}\sum_{j=1}^{3} c_i c_j s_{ij}$, $\widehat{Var}(\hat{b}(\sigma,\tau)) = \sum_{i=4}^{9}\sum_{j=4}^{9} c_i c_j s_{ij}$, and $\widehat{Cov}(\hat{a}(\sigma,\tau), \hat{b}(\sigma,\tau)) = \sum_{i=1}^{3}\sum_{j=4}^{9} c_i c_j s_{ij}$, where $c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9) = (1, \sigma, \tau^{-1}, 1, \sigma, \sigma^2, \tau^{-1}, \tau^{-2}, \sigma\tau^{-1})$. Subsequent formulas for estimates of the variances of $\hat{y}_p(\sigma,\tau)$ and $\hat{R}_t(\sigma,\tau)$, and for associated confidence sets for $y_p(\sigma,\tau)$ and $R_t(\sigma,\tau)$, are identical to those presented in Appendix A, with (σ,τ) substituted for σ throughout.

G. Appendix C

Results analogous to those found in Appendices A and B are presented here to facilitate computation of quantities introduced in part C.

Table 28 gives the burst pressure data for the seven spools used in this study. (Spool 7 here refers to the old spool 8. The original spool 7 data are omitted.)

Tables 29 through 33 give the vessel lifetime data.

The natural logarithm of the ML estimate $\hat{t}_p(\sigma)$ is expressible as
$\hat{y}_p(\sigma) = (\hat{\theta}_1 w_p + \hat{\theta}_9 + \hat{\theta}_3\delta_1 + \hat{\theta}_4\delta_2 + \hat{\theta}_5\delta_3 + \hat{\theta}_6\delta_4 + \hat{\theta}_7\delta_5 + \hat{\theta}_8\delta_6) + \hat{\theta}_{10}\sigma + \hat{\theta}_2 w_p e^{2\sigma}$.

The joint probability distribution of the ML estimates $(\hat{\theta}_1, \ldots, \hat{\theta}_{10})$ is approximately 10-dimensional multivariate normal with mean vector $(\theta_1, \ldots, \theta_{10})$ and covariance matrix Σ estimated by $\hat{\Sigma} = (s_{ij})$, i,j = 1, ..., 10,

$$
= \begin{bmatrix}
1.33 \times 10^{-2} & -4.59 \times 10^{-6} & 1.81 \times 10^{-3} & -2.95 \times 10^{-3} & -3.84 \times 10^{-3} & 7.24 \times 10^{-3} & -4.09 \times 10^{-3} & -1.75 \times 10^{-3} & 2.15 \times 10^{-3} & -4.65 \times 10^{-4} \\
-4.59 \times 10^{-6} & 2.12 \times 10^{-9} & -4.31 \times 10^{-7} & 1.16 \times 10^{-6} & 1.34 \times 10^{-6} & -2.52 \times 10^{-6} & 1.12 \times 10^{-6} & 3.32 \times 10^{-7} & 4.10 \times 10^{-6} & -1.25 \times 10^{-6} \\
1.81 \times 10^{-3} & -4.31 \times 10^{-7} & 4.18 \times 10^{-2} & -4.48 \times 10^{-3} & -7.89 \times 10^{-3} & -8.01 \times 10^{-3} & -8.46 \times 10^{-3} & -6.58 \times 10^{-3} & 1.95 \times 10^{-2} & -4.91 \times 10^{-3} \\
-2.95 \times 10^{-3} & 1.16 \times 10^{-6} & -4.48 \times 10^{-3} & 2.26 \times 10^{-2} & -2.46 \times 10^{-3} & -7.33 \times 10^{-3} & -2.75 \times 10^{-3} & -1.87 \times 10^{-3} & 4.11 \times 10^{-3} & -1.89 \times 10^{-3} \\
-3.84 \times 10^{-3} & 1.34 \times 10^{-6} & -7.89 \times 10^{-3} & -2.46 \times 10^{-3} & 3.76 \times 10^{-2} & -1.21 \times 10^{-2} & -4.48 \times 10^{-3} & -4.15 \times 10^{-3} & -1.92 \times 10^{-2} & 5.07 \times 10^{-3} \\
7.24 \times 10^{-3} & -2.52 \times 10^{-6} & -8.01 \times 10^{-3} & -7.33 \times 10^{-3} & -1.21 \times 10^{-2} & 5.82 \times 10^{-2} & -1.35 \times 10^{-2} & -9.88 \times 10^{-3} & 6.95 \times 10^{-2} & -1.76 \times 10^{-2} \\
-4.09 \times 10^{-3} & 1.12 \times 10^{-6} & -8.46 \times 10^{-3} & -2.75 \times 10^{-3} & -4.48 \times 10^{-3} & -1.35 \times 10^{-2} & 3.97 \times 10^{-2} & -3.83 \times 10^{-3} & -3.94 \times 10^{-2} & 1.05 \times 10^{-2} \\
-1.75 \times 10^{-3} & 3.32 \times 10^{-7} & -6.58 \times 10^{-3} & -1.87 \times 10^{-3} & -4.15 \times 10^{-3} & -9.88 \times 10^{-3} & -3.83 \times 10^{-3} & 3.13 \times 10^{-2} & -2.30 \times 10^{-2} & 5.87 \times 10^{-3} \\
2.15 \times 10^{-3} & 4.10 \times 10^{-6} & 1.95 \times 10^{-2} & 4.11 \times 10^{-3} & -1.92 \times 10^{-2} & 6.95 \times 10^{-2} & -3.94 \times 10^{-2} & -2.30 \times 10^{-2} & 1.18 \times 10^{0} & -3.11 \times 10^{-1} \\
-4.65 \times 10^{-4} & -1.25 \times 10^{-6} & -4.91 \times 10^{-3} & -1.89 \times 10^{-3} & 5.07 \times 10^{-3} & -1.76 \times 10^{-2} & 1.05 \times 10^{-2} & 5.87 \times 10^{-3} & -3.11 \times 10^{-1} & 8.28 \times 10^{-2}
\end{bmatrix}
$$

The corresponding estimates of the variances of the estimators $\hat{a}(\sigma)$ and $\hat{b}(\sigma)$, and of the covariance between $\hat{a}(\sigma)$ and $\hat{b}(\sigma)$, are

$\widehat{\mathrm{Var}}(\hat{a}(\sigma)) = \sum_{i=1}^{2} \sum_{j=1}^{2} c_i c_j s_{ij}$, $\widehat{\mathrm{Var}}(\hat{b}(\sigma)) = \sum_{i=3}^{10} \sum_{j=3}^{10} c_i c_j s_{ij}$, and

$\widehat{\mathrm{Cov}}(\hat{a}(\sigma), \hat{b}(\sigma)) = \sum_{i=1}^{2} \sum_{j=3}^{10} c_i c_j s_{ij}$, where $(c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) = (1, e^{2\sigma}, \delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, 1, \sigma)$. Subsequent formulas for estimates of the variances of $\hat{y}_p(\sigma)$ and $\hat{R}_t(\sigma)$, and for associated confidence sets for $y_p(\sigma)$ and $R_t(\sigma)$ are precisely as presented in Appendix A.

Table 28. Burst pressure, in MPa, of NASA vessels.

Spool 1	Spool 2	Spool 3	Spool 4	Spool 5	Spool 6	Spool 7
33.09	32.75	32.47	36.03	31.99	34.47	34.13
34.89	33.72	34.54	37.23	33.72	34.75	
35.32	33.85	34.99		34.96		
35.58	33.96	35.16				
35.68	35.58	36.82				
36.54						

TABLE 29.
Failure Times (hr) for NASA Vessels
Pressure: 4.28 ksi
Temperature: 25 deg C.
Number of specimens: 31

Spool 1	Spool 2	Spool 3	Spool 4	Spool 5	Spool 6	Spool 7
444.4	2.2	12.5	254.1	8.3	6.7	98.2
755.2	8.5	14.6	1148.5	13.3	15.0	590.1
952.2	9.1	18.7	1569.3	87.5	144.0	638.2
1108.2	10.2	101.0	1750.6	243.9		
	22.1		1802.1			
	55.4					
	111.4					
	158.7					

TABLE 30.
Failure Times (hr) for NASA Vessels
Pressure: 3.97 ksi
Number of specimens: 24

Spool 1	Spool 2	Spool 3	Spool 4	Spool 5	Spool 6	Spool 7
453.4	71.2	19.1	876.7	541.6	514.2	554.2
664.5	199.1	24.3	1275.6		1254.9	2046.2
930.4	403.7	69.8	1536.8			
1755.5	453.4	136.0	6177.5			
	514.1					
	544.9					
	694.1					

TABLE 31.
Failure Times (hr) for NASA Vessels
Pressure: 3.68 ksi
Temperature: 25 deg C.
Number of specimens: 23

Spool 1	Spool 2	Spool 3	Spool 4	Spool 5	Spool 6	Spool 7
11487.3	1134.3	1087.7	13501.3	11727.1	225.2	2974.6
14032.	1824.3	2442.5	29808.		6271.1	g
30977.	1920.1				7996.0	7332.0
	2383.0					7918.7
	3708.9					9240.3
	5556.0					9973.3

g Grouped time in the interval (4872.,4944.).

TABLE 32.
Failure Times (hr) for NASA Vessels
Pressure: 3.38 ksi
Temperature: 25 deg C.
Number of specimens: 21

Spool 1	Spool 2	Spool 3	Spool 4	Spool 5	Spool 6	Spool 7
	14440.	8615.		9120.	7320.	
				20231.	16104.	
				35880.	20233.	

Censored Times and Quantities

Spool 1	Spool 2	Spool 3	Spool 4	Spool 5	Spool 6	Spool 7
44376(4)		3264(1)	192(1)			44376(3)
			44376(4)			

TABLE 33.
Censored Times (hr) and Quantities for NASA Vessels
Pressure: 2.49 ksi
Temperature: 25 deg C.
Number of specimens: 40

Spool 1	Spool 2	Spool 3	Spool 4	Spool 5	Spool 6	Spool 7
44544(2)	44544(1)	44688(3)	44688(7)	44544(1)	44688(7)	1416(1)
44688(3)	44688(5)			44688(7)		44688(3)

H. Figures

Figures 1 through 19 display many of the statistical inferences presented in parts A, B, C, and D.

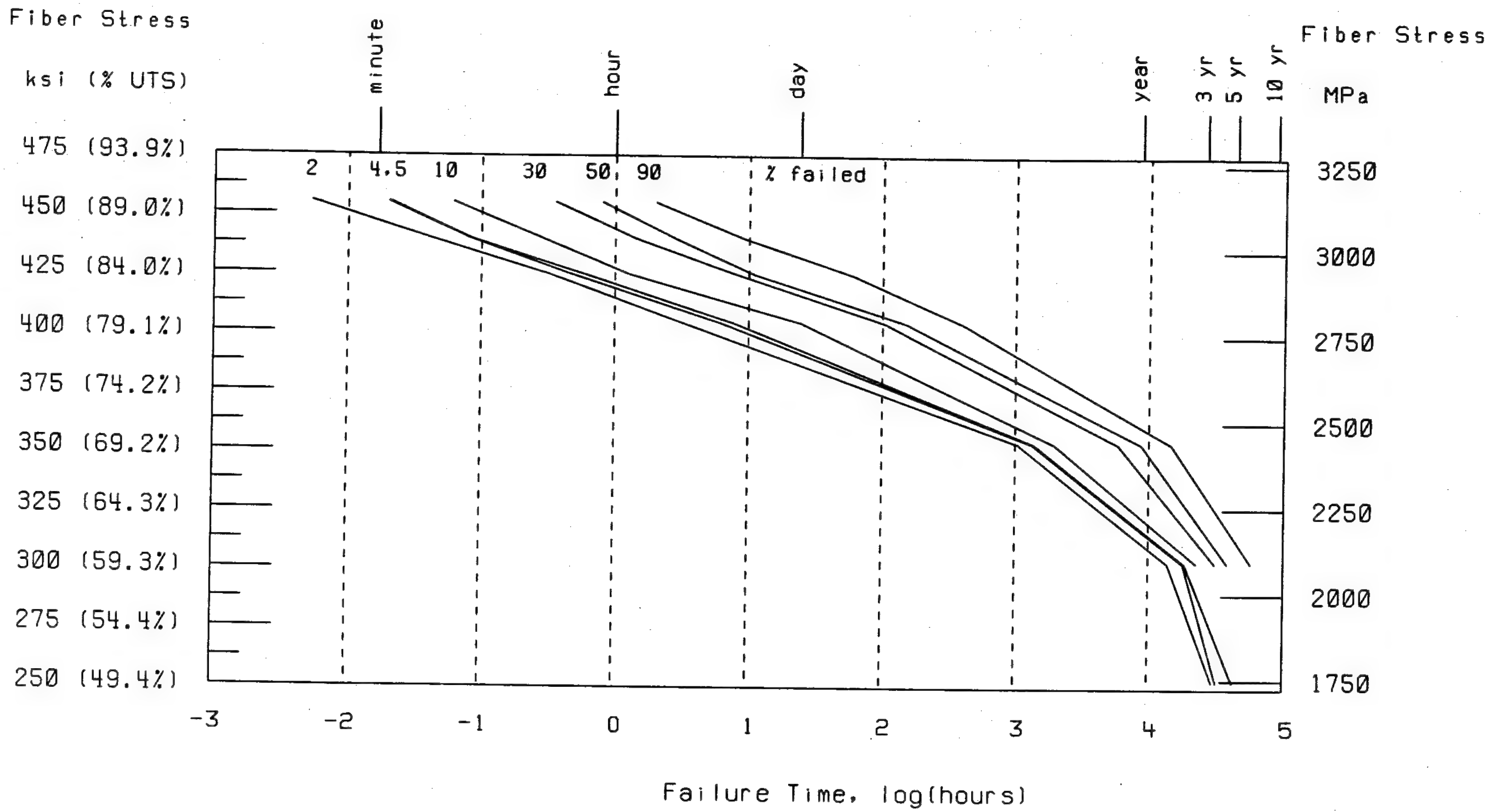


Figure 1. Experimental eight-year lifetime data of Kevlar/epoxy strands (Room temperature, UV).

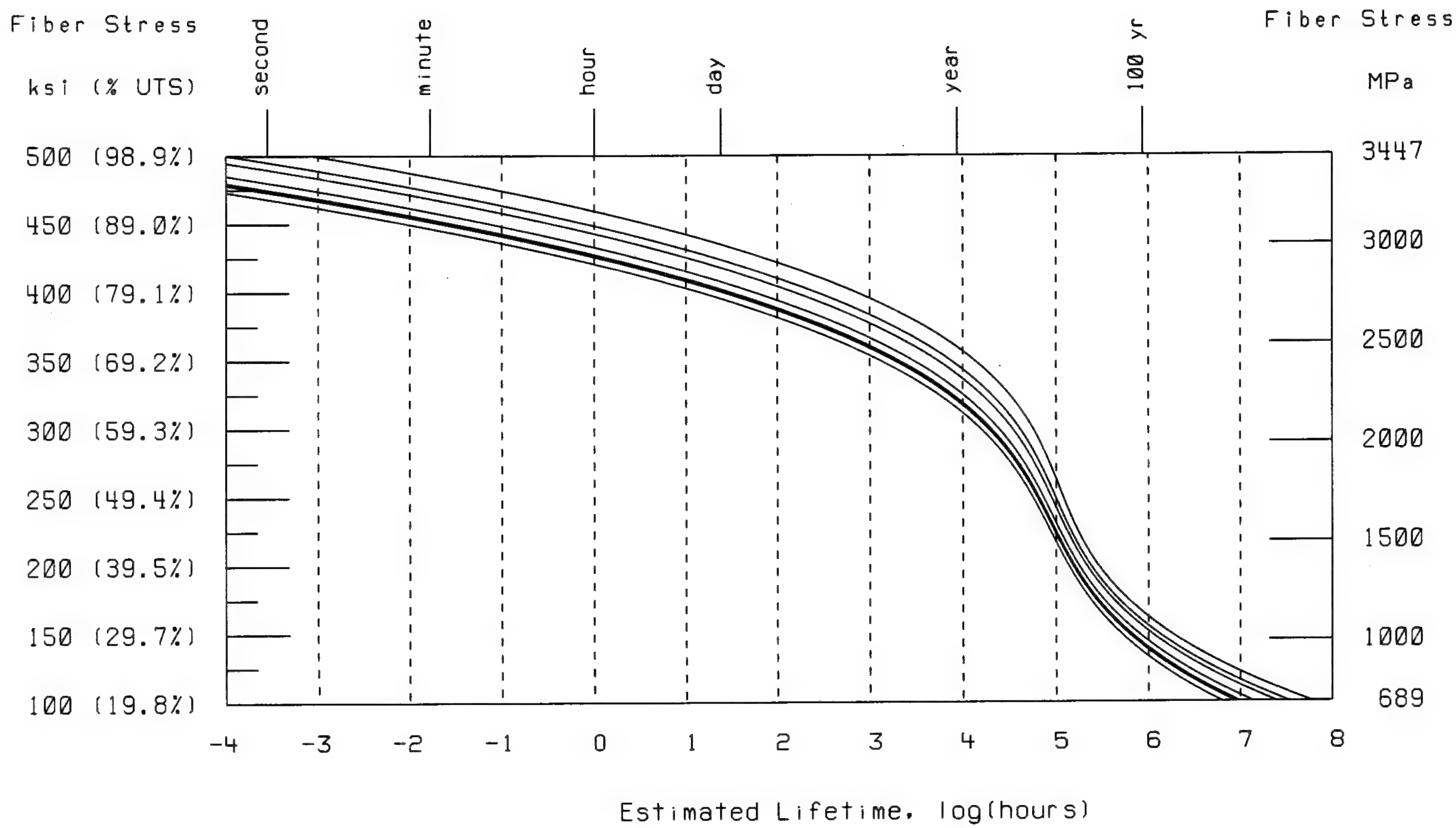


Figure 2. Maximum likelihood estimates of Kevlar/epoxy strands for percentile failure probabilities (left to right) of 2, 4, 5, 10, 30, 50, and 90 (Room temperature, UV).

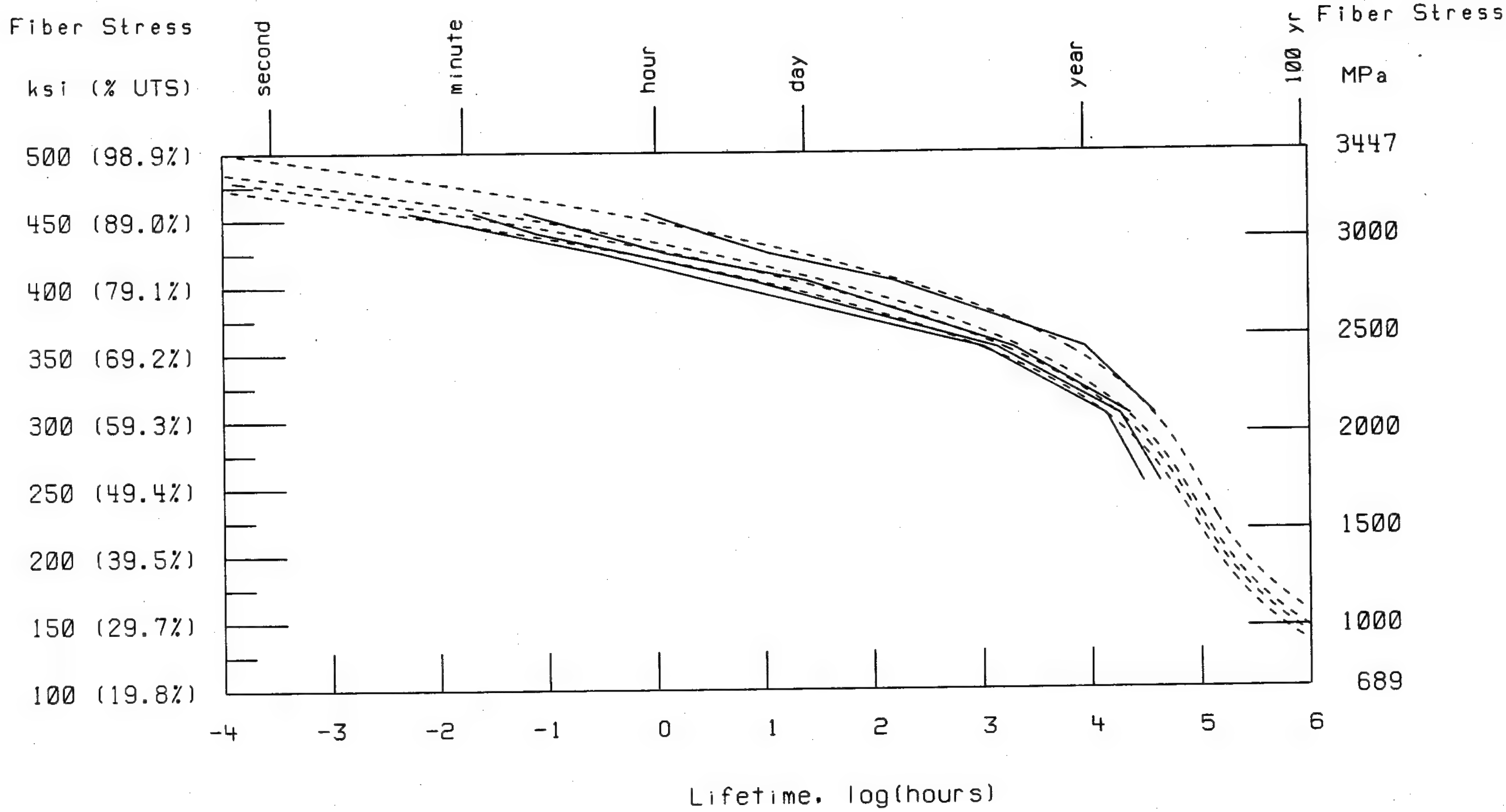


Figure 3. Comparison of experimental data on Kevlar/epoxy strands vs. maximum likelihood lifetime estimates for percentile failure probabilities (left to right) to 2, 5, 10, and 50 (room temperature, UV).

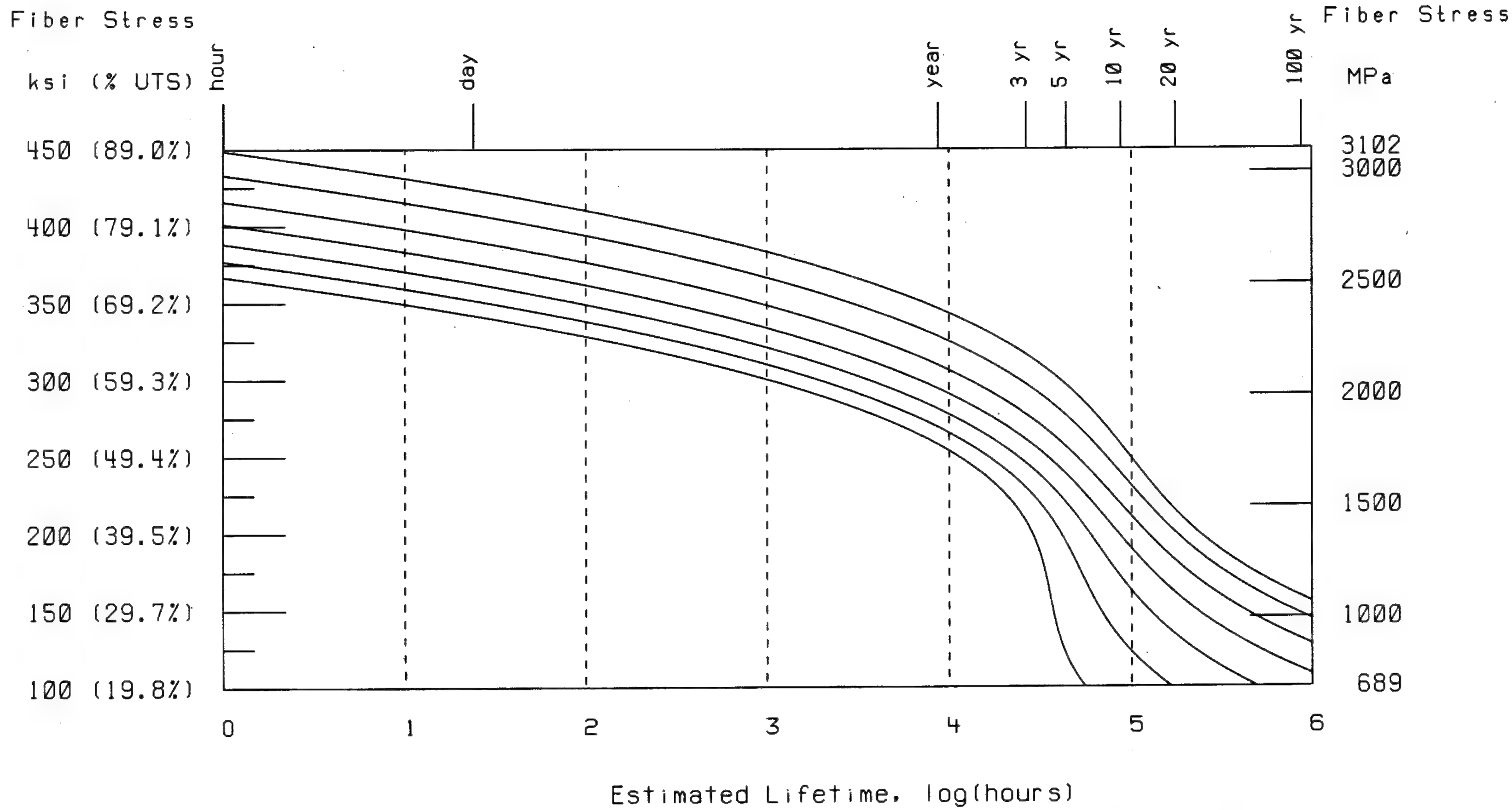


Figure 4. Maximum likelihood estimates of lifetimes of Kevlar/epoxy strands for quantile probabilities (left to right) of 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, and .50 (room temperature, UV).

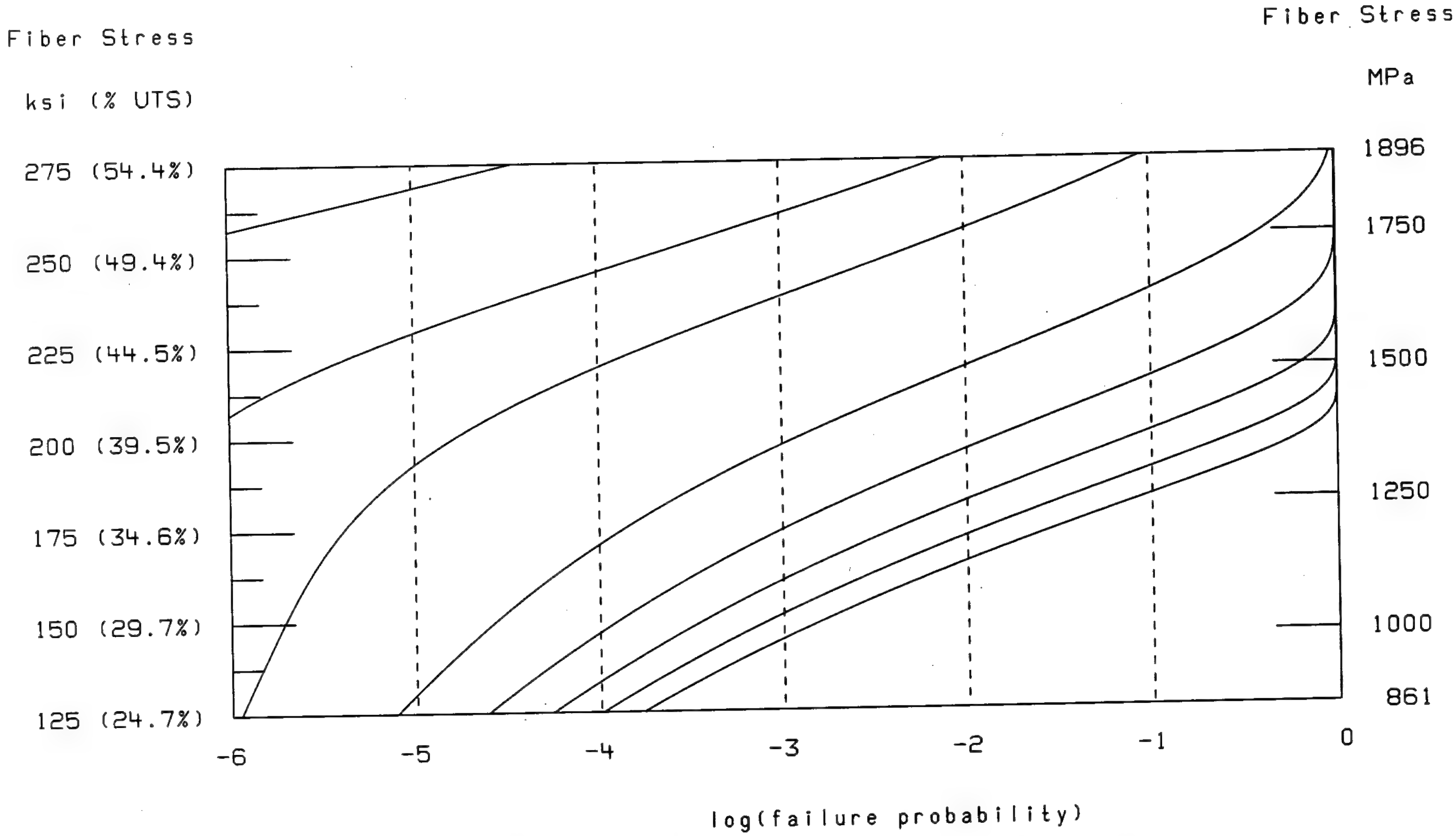
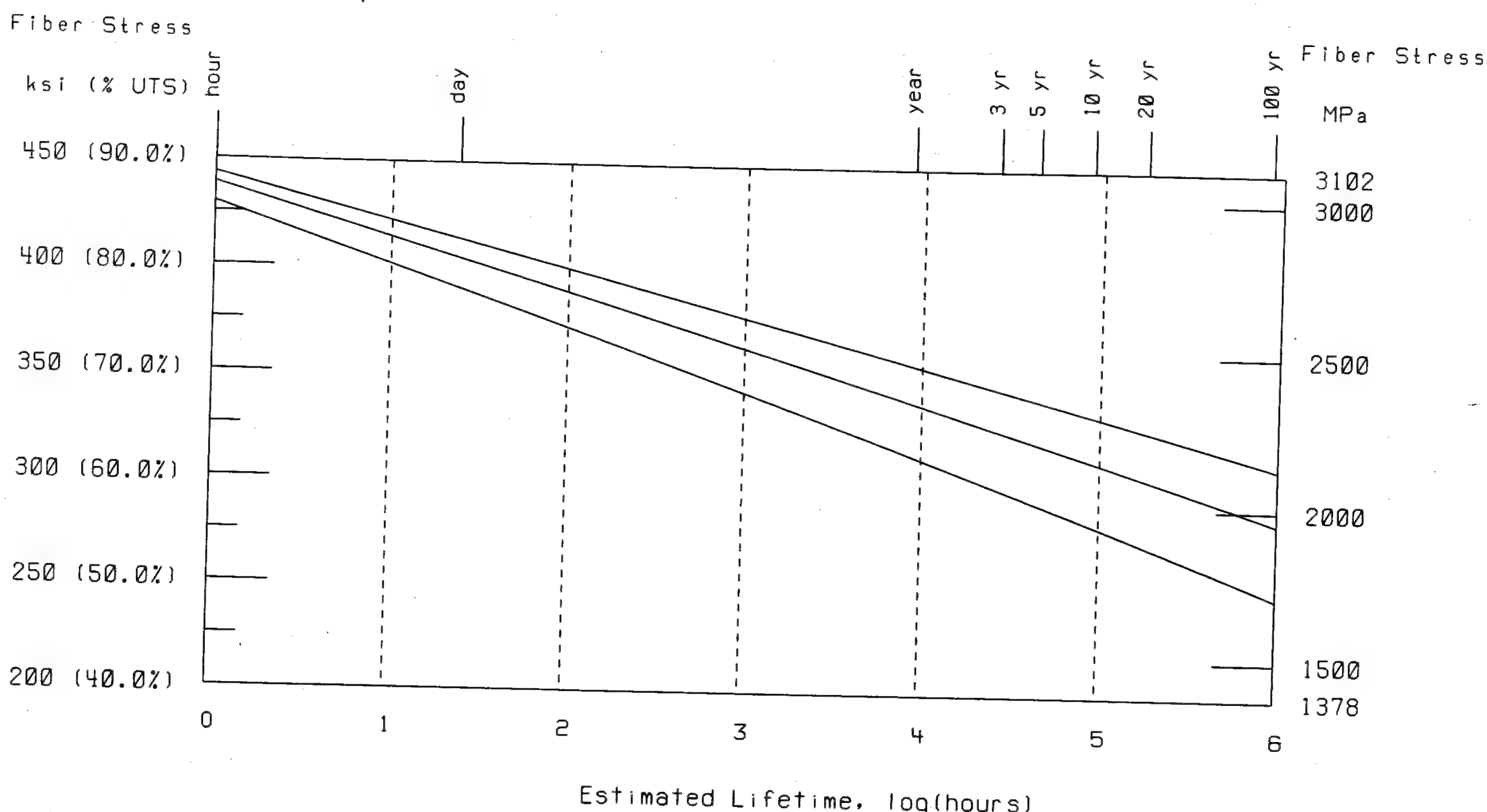


Figure 5. Maximum likelihood estimates of failure probabilities at times (left to right) 1, 3, 5, 10, 15, 20, 25, and 30 years for Kevlar/epoxy strands under tension (room temperature, UV).



Quantile: 5.0e-01
Temperatures (C.): 65.0 45.0 25.0

Figure 6. ML estimates of lifetimes for Kevlar/epoxy strands under tension.

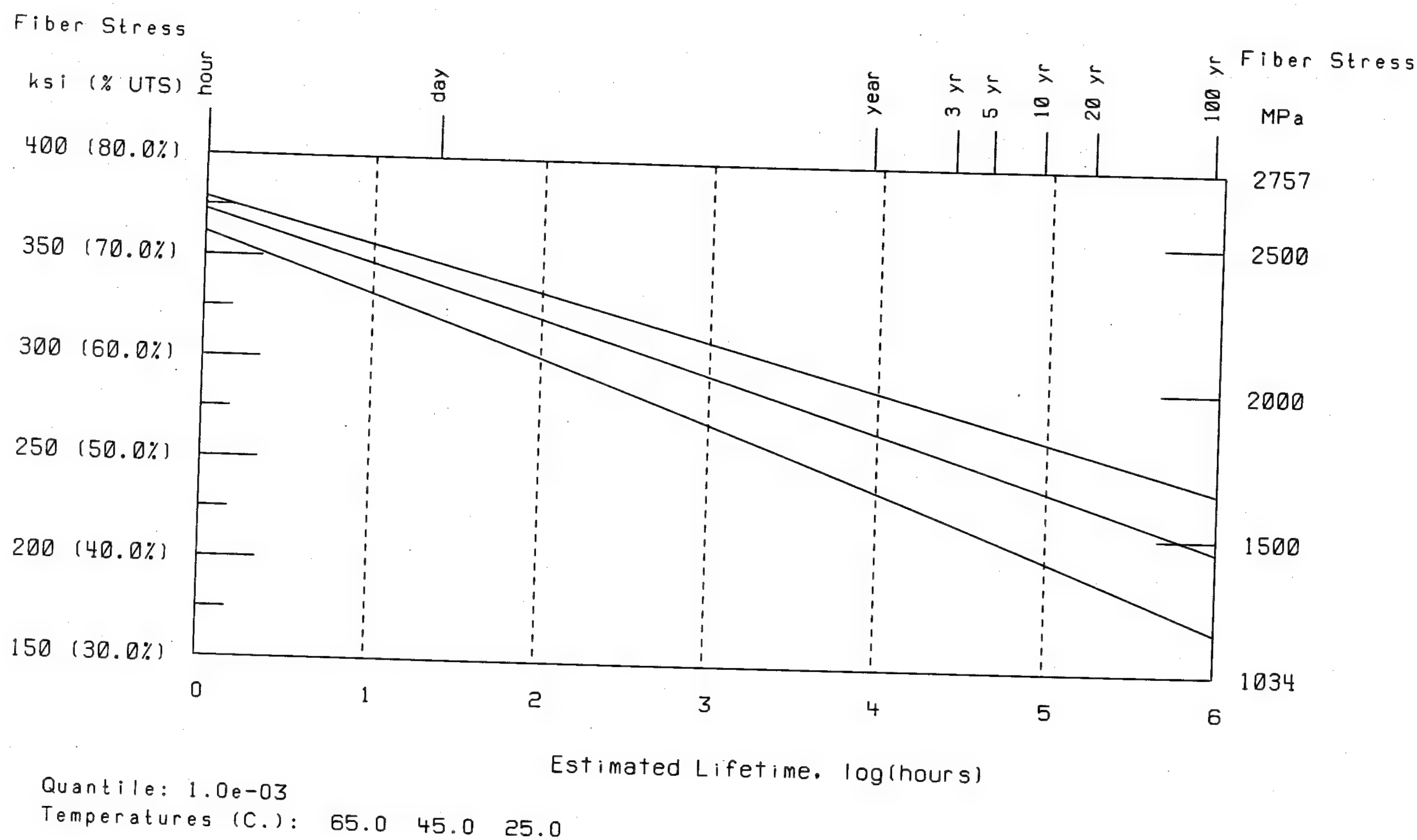
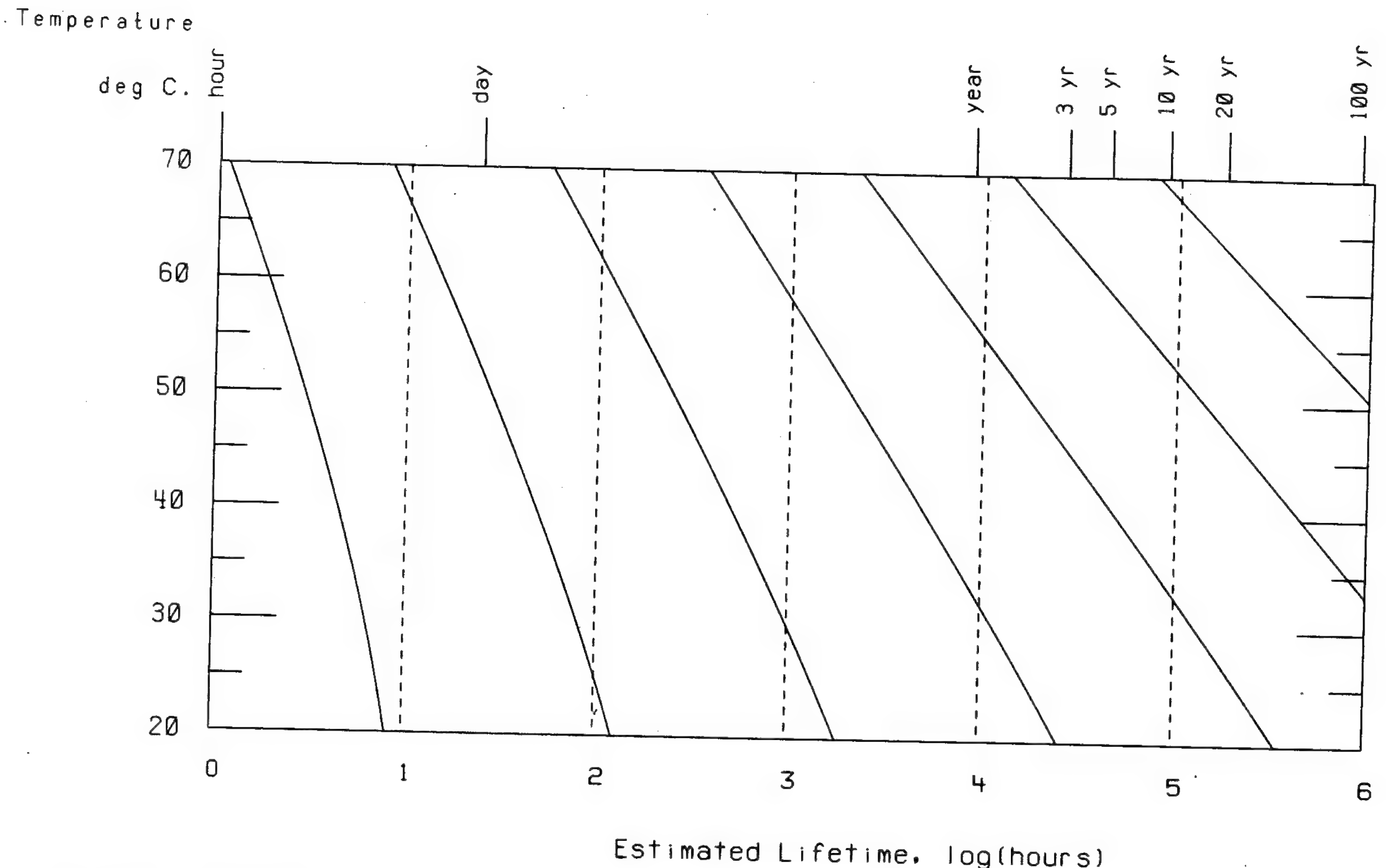
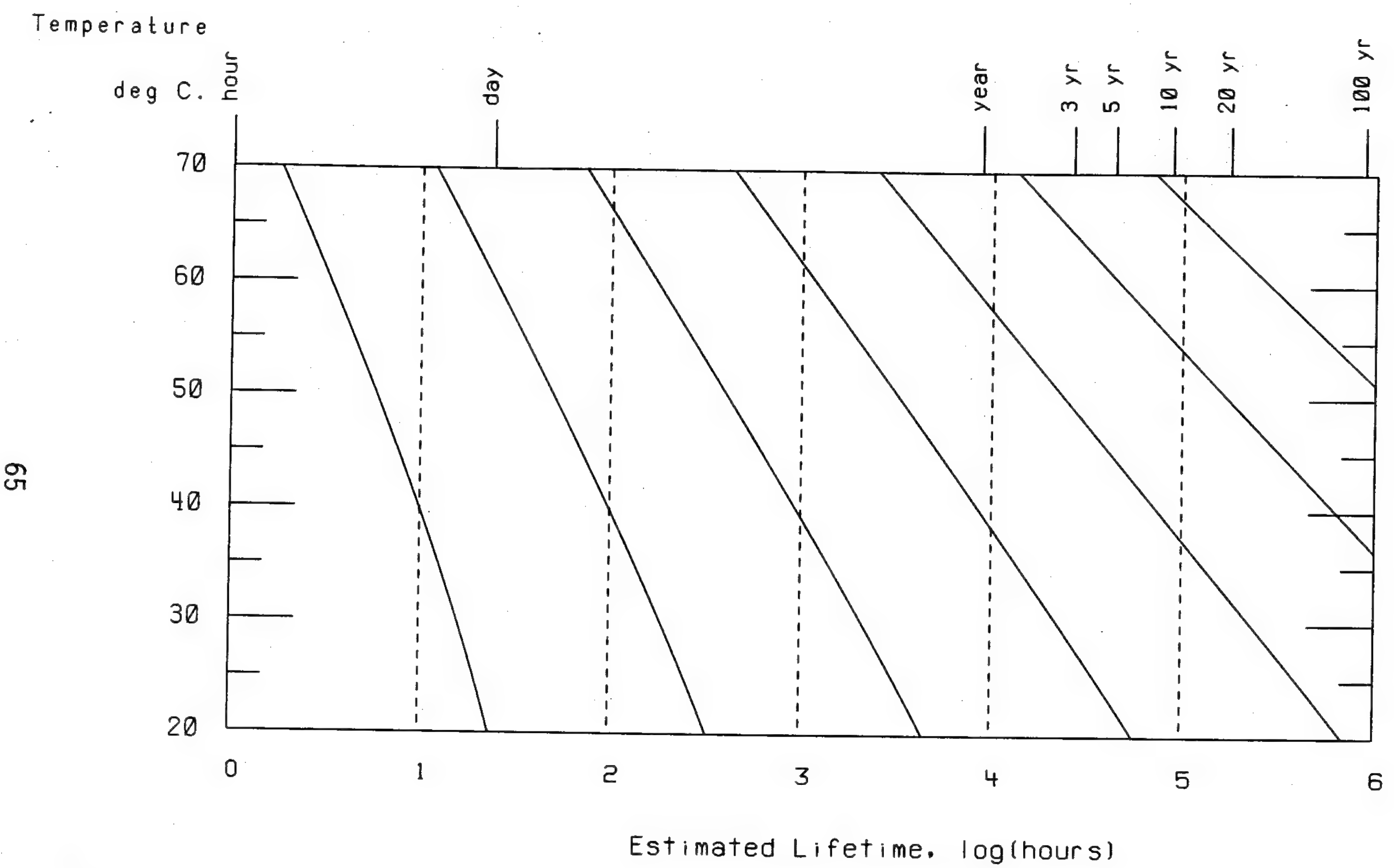


Figure 7. ML estimates of lifetimes for Kevlar/epoxy strands under tension.



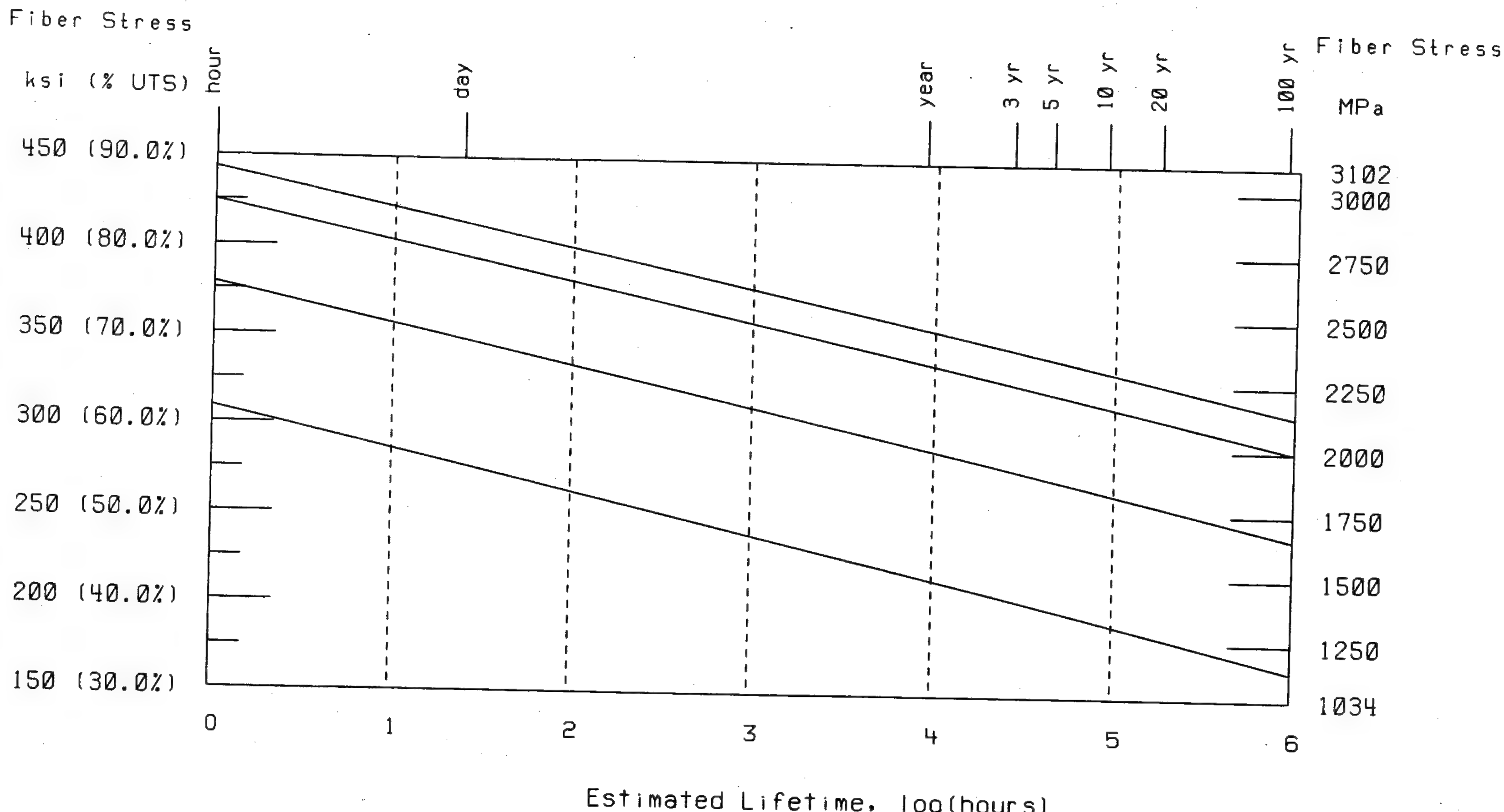
Quantile: 5.0e-01
Fiber Stresses, ksi (UTS=500.): 425. 400. 375. 350. 325. 300. 275.

Figure 8. ML estimates of lifetimes for Kevlar/epoxy strands under tension.



Quantile: 1.0e-03
Fiber Stresses, ksi. (UTS=500.): 350. 325. 300. 275. 250. 225. 200.

Figure 9. ML estimates of lifetimes for Kevlar/epoxy strands under tension.



Temperature: 25.0 degrees C.
Quantiles: 1.e-06 1.e-03 1.e-01 5.e-01

Figure 10. ML estimates of lifetimes for Kevlar/epoxy strands under tension.

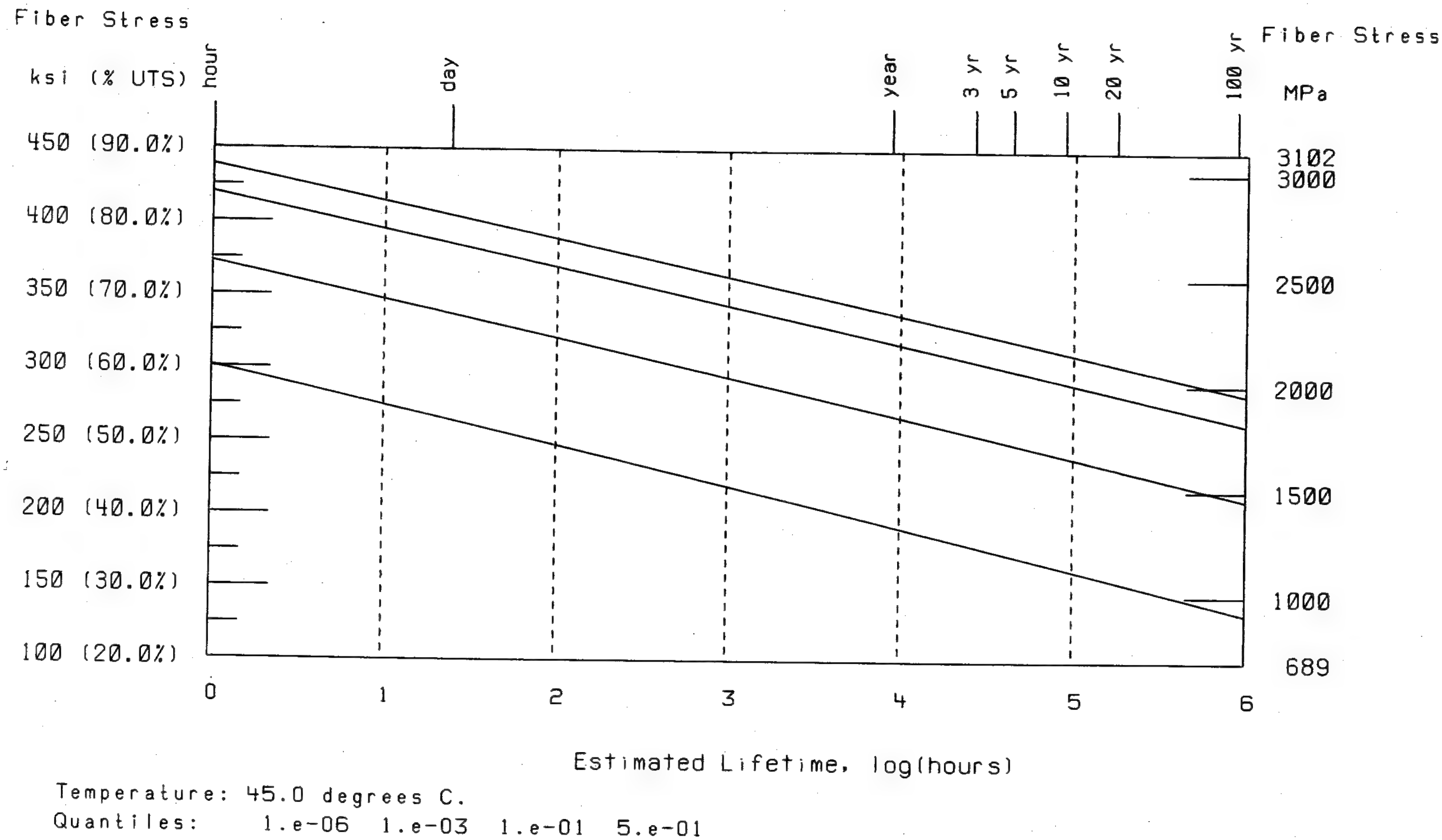
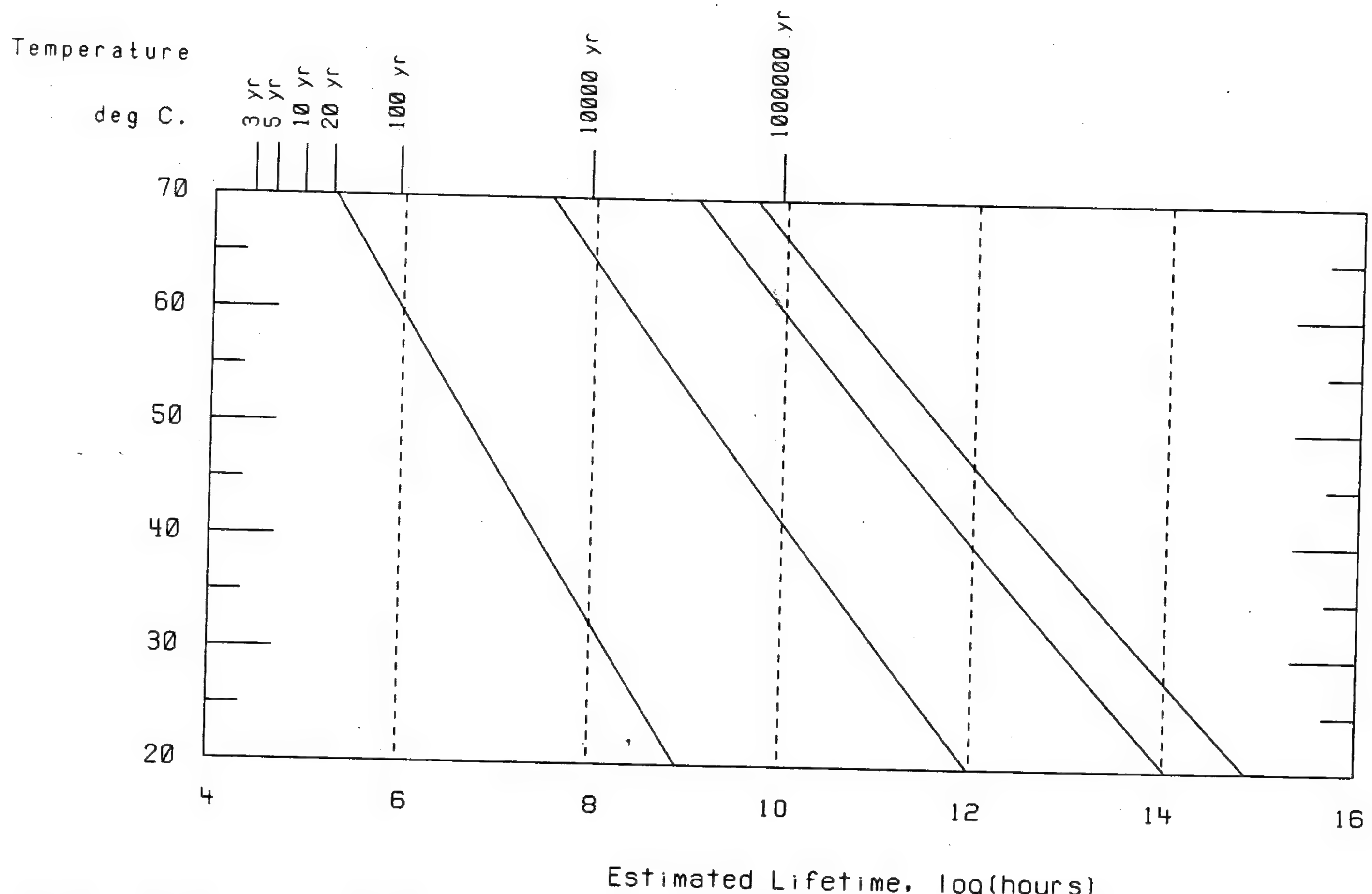
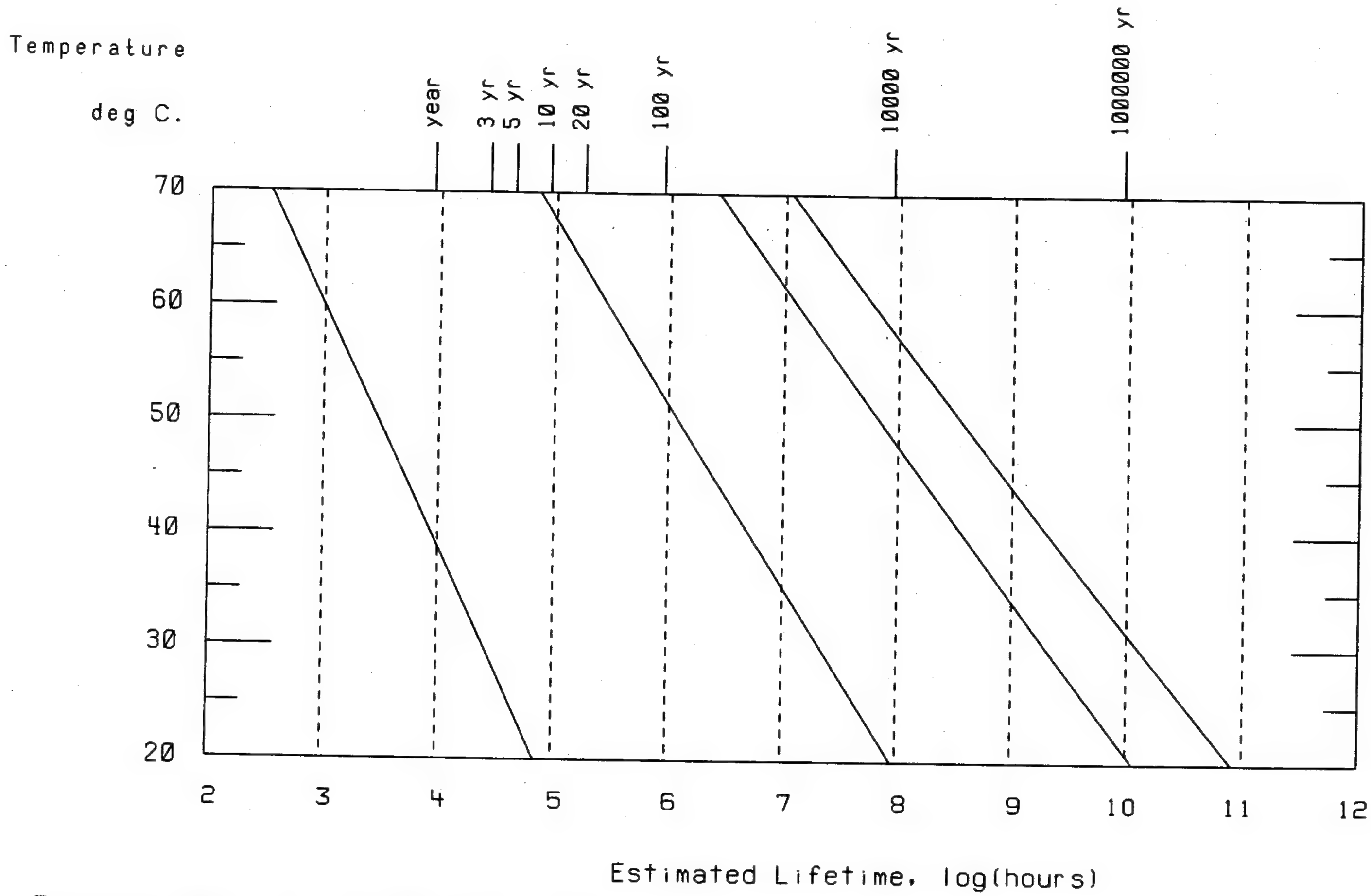


Figure 11. ML estimates of lifetimes for Kevlar/epoxy strands under tension.



Fiber Stress, ksi (UTS=500.): 100.
Quantiles: 1.e-06 1.e-03 1.e-01 5.e-01

Figure 12. ML estimates of lifetimes for Kevlar/epoxy strands under tension.



Fiber Stress, ksi (UTS=500.): 200.
Quantiles: 1.e-06 1.e-03 1.e-01 5.e-01

Figure 13. ML estimates of lifetimes for Kevlar/epoxy strands under tension.

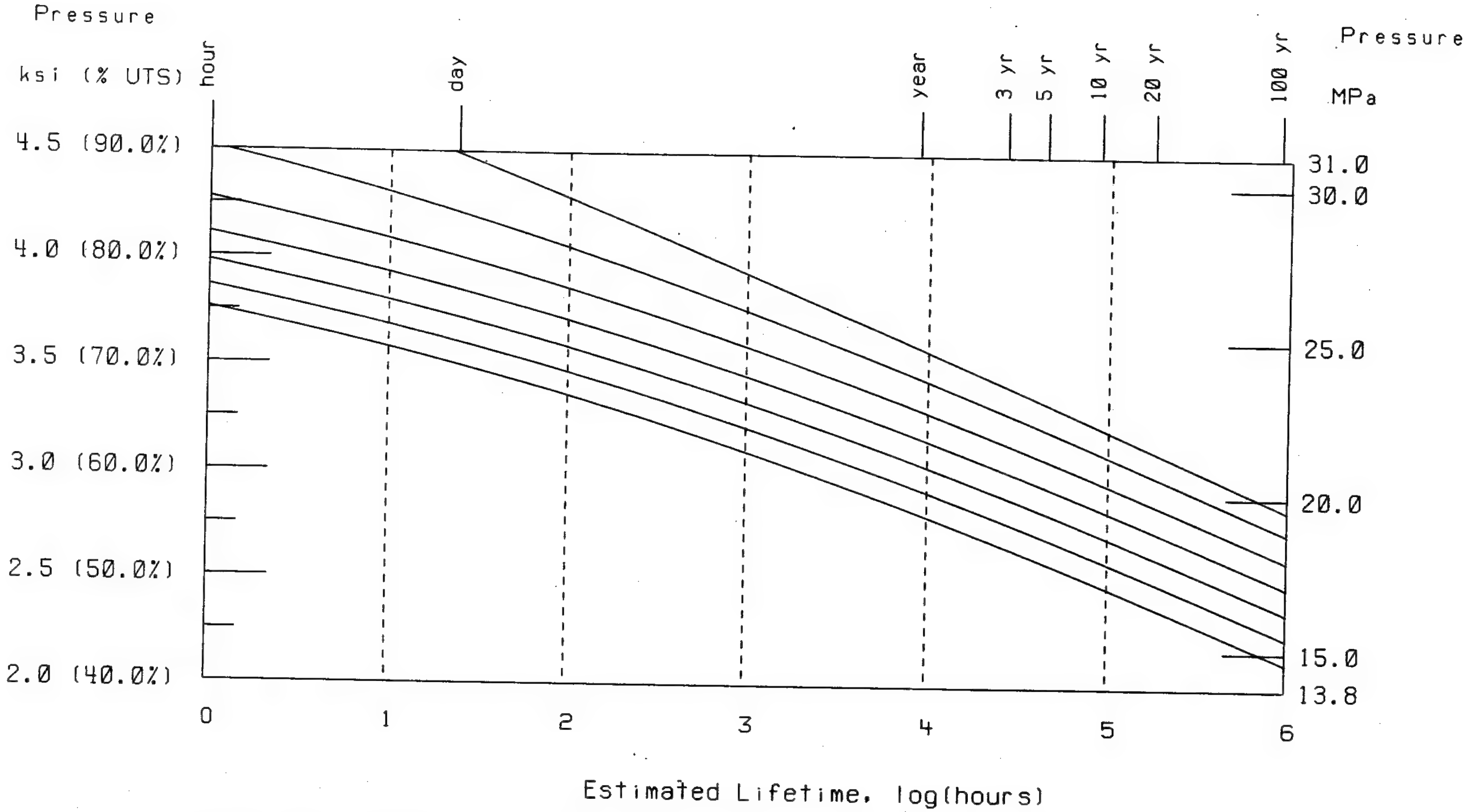


Figure 14. NASA Pressure vessels (room temperature, No UV). Maximum likelihood estimated lifetime quantiles for (left to right) 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, and .50 failure probabilities, based on average spool effect.

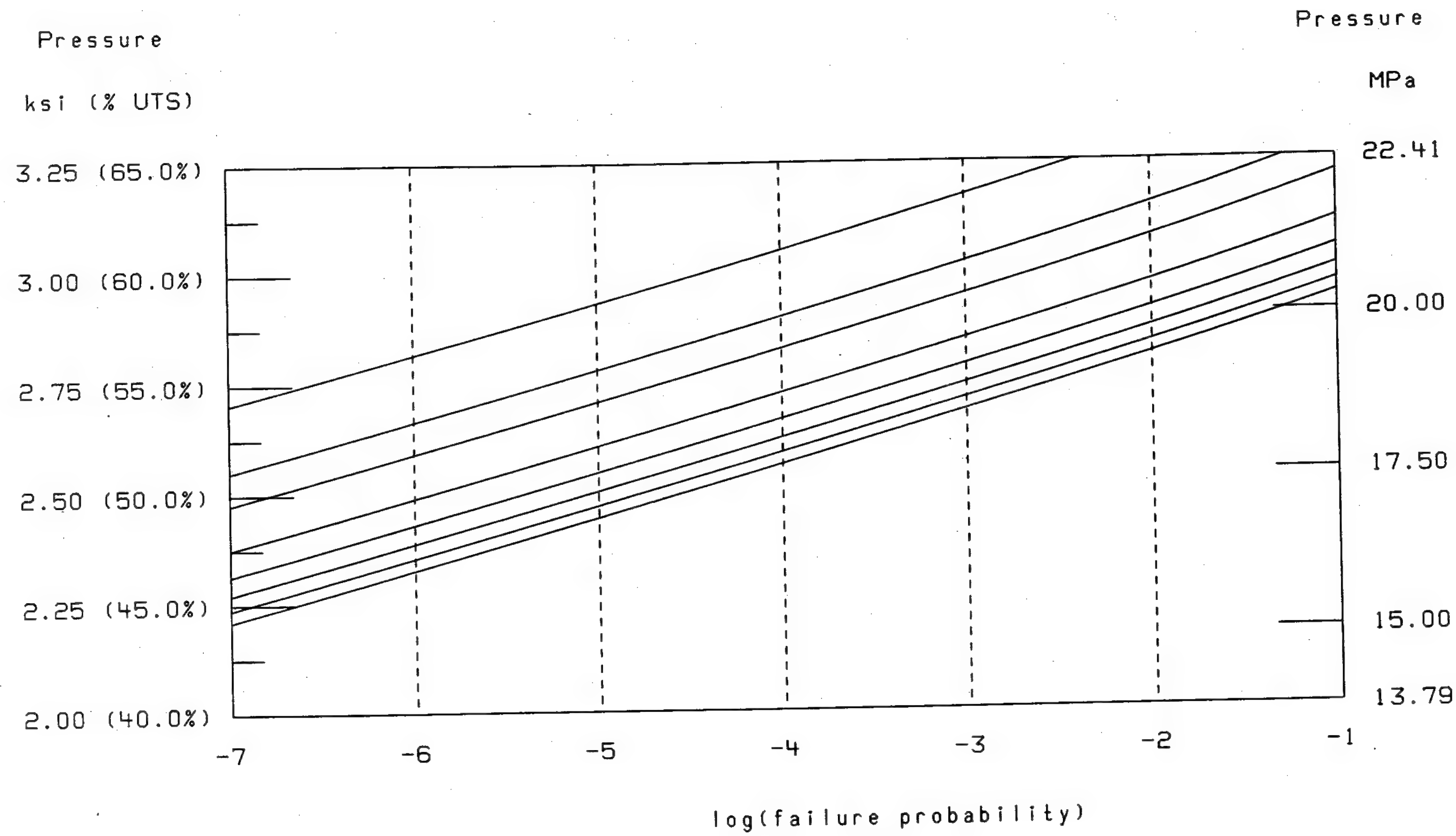


Figure 15. NASA pressure vessels (room temperature, No UV). Maximum likelihood estimated failure probabilities at times (top to bottom) 1, 3, 5, 10, 15, 20, 25, and 30 years, based on average spool effect.

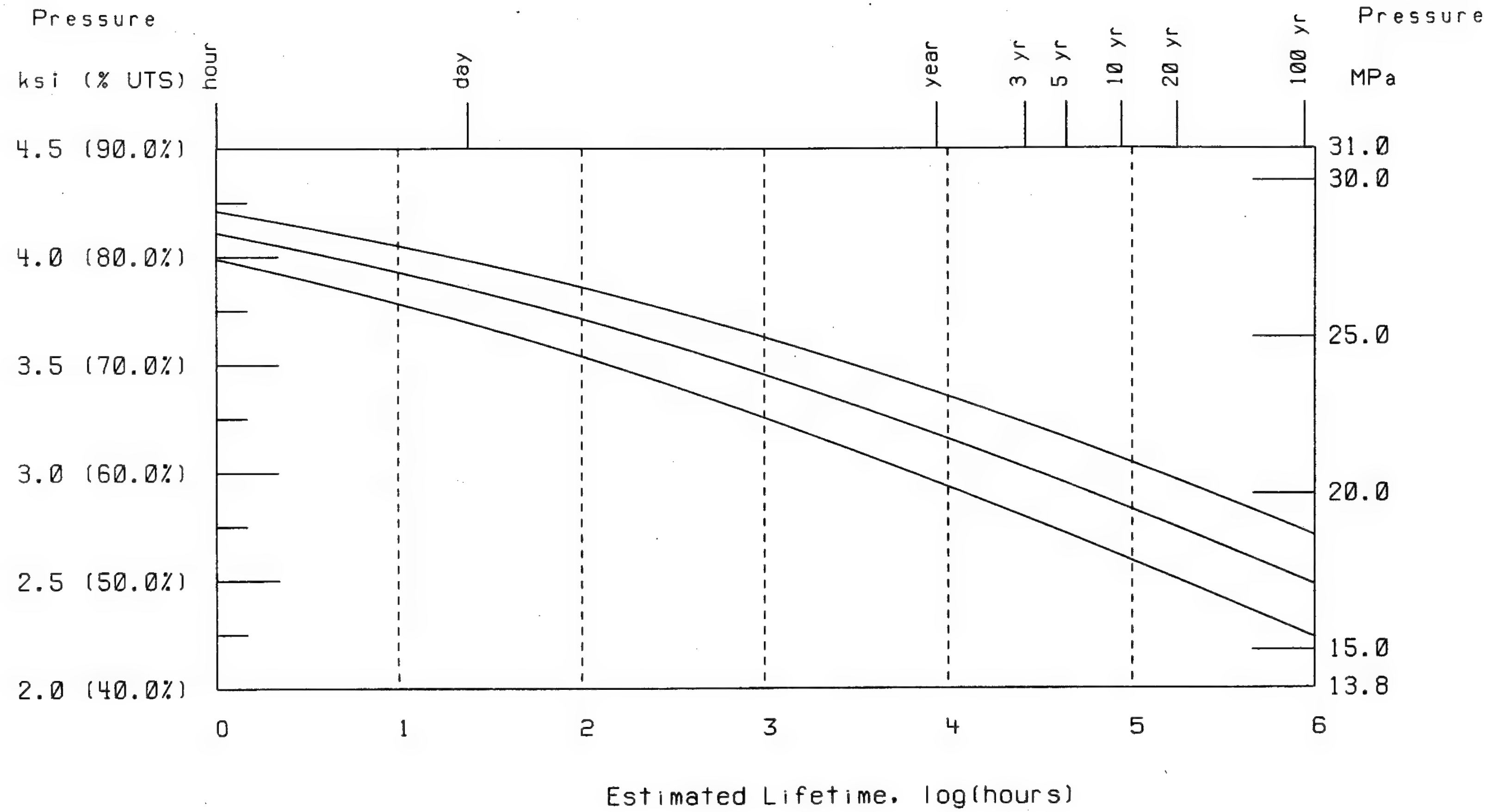


Figure 16. NASA pressure vessels (Room temperature, No UV). Maximum likelihood estimates of 10^{-3} failure probability quantile, based on (top to bottom) best, average, and worst spool effect.

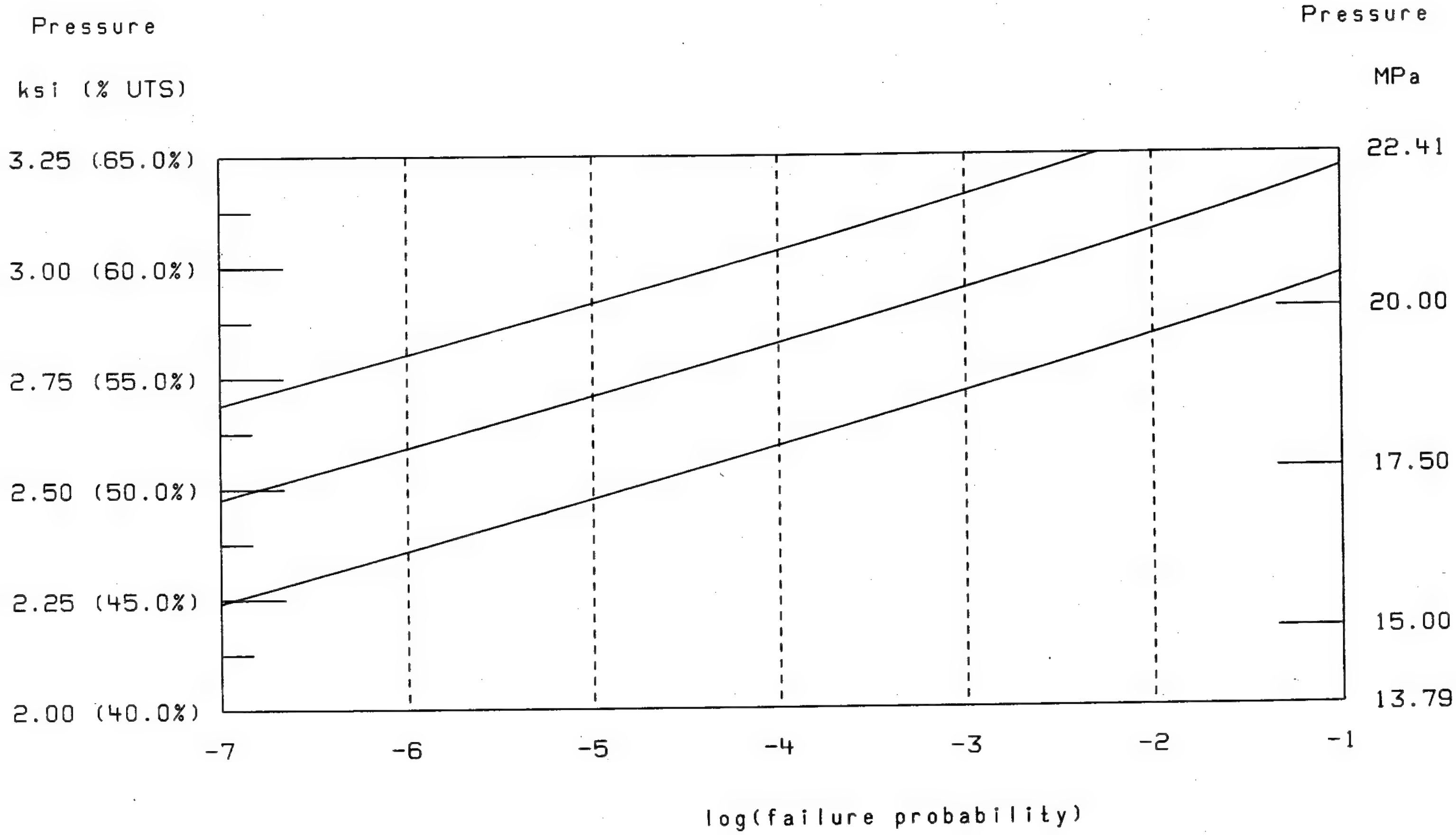


Figure 17. NASA pressure vessels (room temperature, No UV). Maximum likelihood estimates of the failure probability at five years, based on (top to bottom) best, average, and worst spool effect.

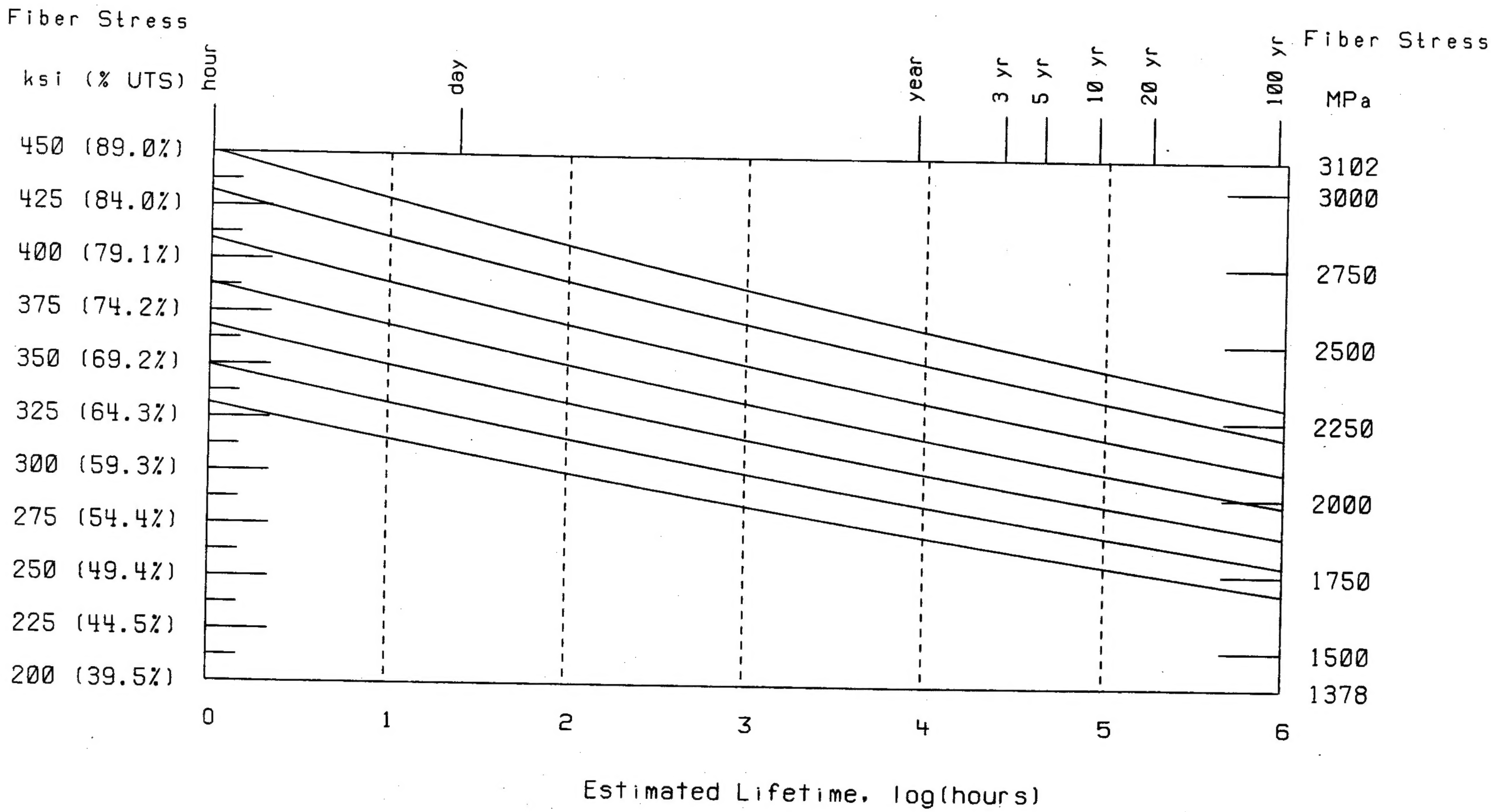


Figure 18. ML estimates of lifetimes for Kevlar/epoxy strands under tension based on data from high stress levels ($\geq$ 80% UTS) and the power law fit. (Room temperature, UV).

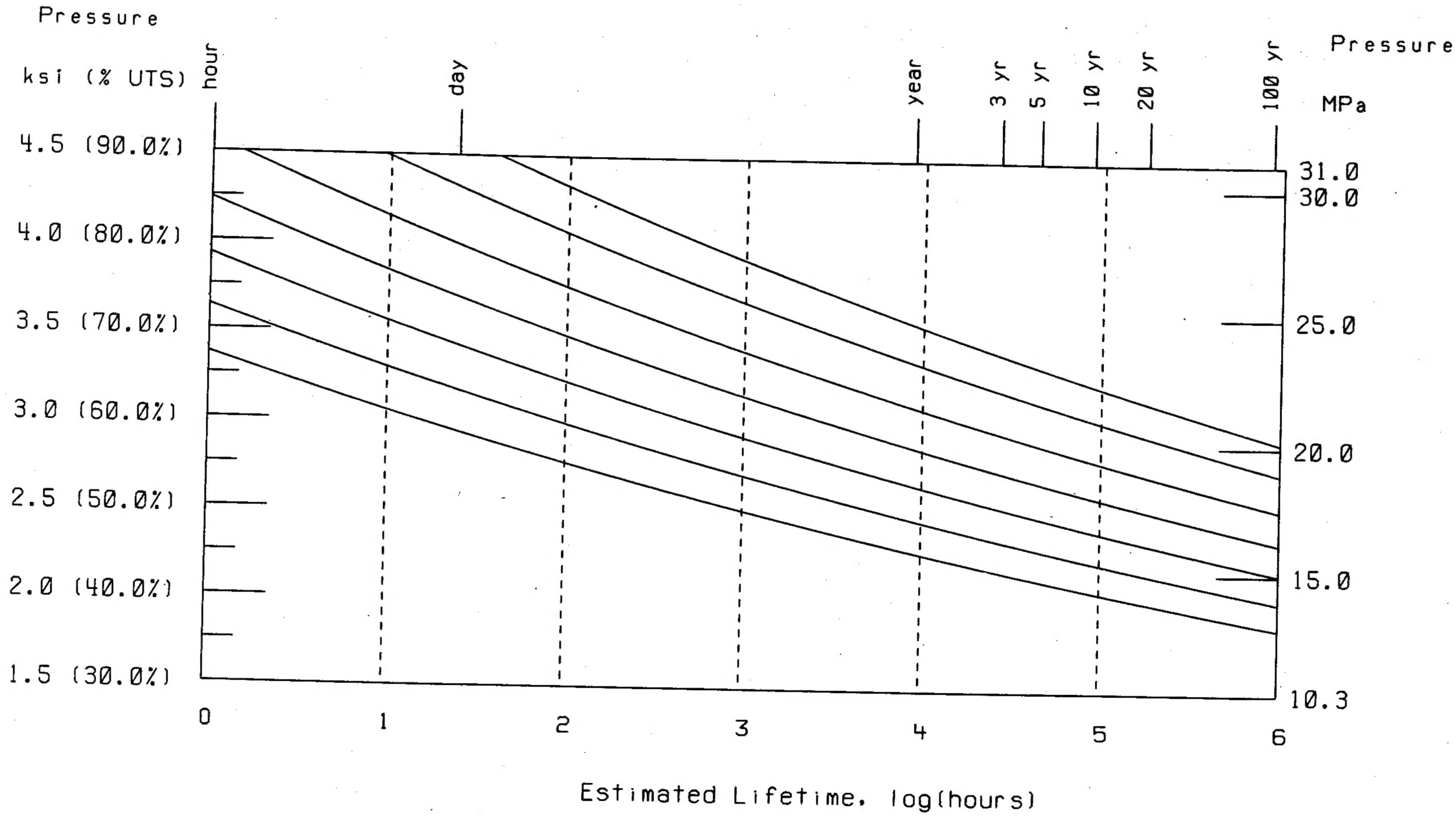


Figure 19. NASA pressure vessels (room temperature, No UV). ML estimated lifetime quantiles for (left to right) 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, and .50 failure probabilities, based on average spool effect and power law fit.

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